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## Review Article

# Simulation Strategies for Mixed Traffic Conditions: A Review of Car-Following Models and Simulation Frameworks

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The area of traffic flow modelling and analysis that bridges civil engineering, computer science, and mathematics has gained significant momentum in the urban areas due to increasing vehicular population causing traffic congestion and accidents. Notably, the existence of mixed traffic conditions has been proven to be a significant contributor to road accidents and congestion. The interaction of vehicles takes place in both lateral and longitudinal directions, giving rise to a two-dimensional (2D) traffic behaviour. This behaviour contradicts with the traditional car-following (CF) or one-dimensional (1D) lane-based traffic flow. Existing one-dimensional CF models did the inclusion of lane changing and overtaking behaviour of the mixed traffic stream with specific alterations. However, these parameters cannot describe the continuous lateral manoeuvre of mixed traffic flow. This review focuses on all the significant contributions made by 2D models in evaluating the lateral and longitudinal vehicle behaviour simultaneously. The accommodation of vehicle heterogeneity into the car-following models (homogeneous traffic models) is discussed in detail, along with their shortcomings and research gaps. Also, the review of commercially existing microscopic traffic simulation frameworks built to evaluate real-world traffic scenario are presented. This review identified various vehicle parameters adopted by existing CF models and whether the current 2D traffic models developed from CF models effectively captured the vehicle behaviour in mixed traffic conditions. Findings of this study are outlined at the end.

## 1. Introduction

In transport engineering, road safety and traffic flow congestion have received increased attention in the past few years. The most common and highlighted problems of any urban area are traffic congestion and road accidents. A large number of road accidents and traffic jams produce a significant impact on the overall gross domestic product (GDP) of a country [1, 2]. This is shown in Table 1, based on the UNESCAP report 2017. These problems have a severe consequence on urban dwellers. Half of the human population (i.e., 3.5 billion people) lives in cities today, and 5 billion people are estimated to dwell in cities by the year 2030. The World Health Organization (WHO 2016)

reported that nearly 1.3 million people are dying and 50 million people are wounded in traffic accidents every year. The 9<sup>th</sup> leading cause of death is road traffic accidents, which accounts for 2.2% of all deaths globally. Young adults between the ages of 15 and 44 constitute to 59% of accident deaths. Some of the other impacts include uncontrolled motorization and urban poverty. Current projections show that road crashes will become the fifth leading cause of death by the year 2030.

Approximately 316,000 people are killed due to road injuries in the Southeast Asian region on a yearly basis, which makes up 25% of global road traffic deaths. The road traffic death rate in the Southeast Asian region is 17.0 per 100,000, compared to the worldwide scale of 17.4 [3].

TABLE 1: Road safety situation in Southeast Asia (source UNESCAP report 2017).

Estimated losses due to road traffic crashes (2013)	Estimated GDP loss (%)	Estimated loss (million USD)
Bangladesh	1.6	2456.08
India	3	58,082.64
Indonesia	2.9-3.0	22,652.82
Malaysia	1.5	4,697.37
Myanmar	0.5	310.71
Philippines	2.6	7,073.74
Thailand	3	12,605.01
Vietnam	2.9	4,965.44
Cambodia	2.1	324.45

Cyclists and motorcyclists (“vulnerable road users”) make up approximately 50% of road deaths. A large proportion of global road deaths occur in the Southeast Asian region. To achieve the adopted sustainable development goal on road safety of reducing the global rate of traffic deaths and injuries by the year 2020, effective road safety measures must be enforced.

The National Academy of Engineering identified the Grand Challenges for Engineering in the 21<sup>st</sup> century. The 14 game-changing goals were broadcasted in 2008 for improving life on the planet. The 7<sup>th</sup> and most crucial grand challenge is to Restore and Improve Urban Infrastructure. Moreover, one of the United Nations Sustainable Development Goals (UNSDG) is Sustainable Cities and Communities. One of the more significant challenges today for engineers is the integration of traffic systems along with various private and public transport, to achieve an integration of systems paving the way for the development of ITS [4].

The traffic in Southeast Asian countries is a mixture of different kinds of vehicles of varying size, velocity acceleration, etc., giving rise to nonlane behaviour. The absence of lane behaviour causes vehicles to interact frequently in lateral and longitudinal directions with the surrounding vehicles. Therefore, a detailed study needs to be conducted to evaluate the heterogeneous traffic flow induced by different driver behaviours.

Driver behaviour is affected by traffic and roadway features. The influence on the capacity of roadways depends not only on longitudinal behaviour but also on the lateral response of the vehicles in diverse conditions (traffic consisting of vehicles vary in size and type), also known as heterogeneous or no-lane-based traffic. In such varied conditions (conditions where vehicles do not adequately follow lane behaviour), the drivers are left with two primary tasks, namely, control of the vehicle in the longitudinal direction, along with the lateral manoeuvre by choice of steering angles along the width of the roadway. Both tasks are interrelated and co-occur, giving rise to two-dimensional (2D) vehicle manoeuvre. This type of combined driver behaviour is more complicated as compared to car-following and lane-changing models, giving rise to multiple leaders following, abreast manoeuvring, tailgating, filtering, and swerving. Therefore, an in-depth study of vehicle parameters at the microscopic level to evaluate the traffic stream and build a generic numerical model is crucial. At the same time,

a few research works have been carried out by researchers to explain the complex driving nature in heterogeneous traffic flow, also called 2D traffic stream.

The vehicles in heterogeneous traffic conditions do not move along the centre of the lane; this deviation from the centre leads to nonlane vehicle behaviour. All the existing 1D models consider only the longitudinal interactions between cars and thus cannot describe the lateral interactions as in the case of the car-following (CF) theory. These types of models are suitable for homogeneous traffic streams following strict lane behaviour. Hence, an in-depth study in lateral and longitudinal movements of vehicles is necessary to evaluate the driver’s conduct in this heterogeneous traffic flow. So, the models that can describe the combined actions of vehicles can be developed to assess this 2D traffic flow. No widely used theory can comprehensively simulate the 2D traffic flow, i.e., lateral and longitudinal. The past literature that is available on one-dimensional (1D) traffic cannot be directly utilised to model the 2D traffic flow, as there is a wide variation in driver’s behaviour in diverse traffic, as compared to the lane-based traffic stream.

Few studies are available on heterogeneous traffic conditions, especially in developing countries [5]. Limited real-time data formed the foundation for these studies, consisting of convenient assumptions on the movement of vehicles. The previously implemented behavioural models for heterogeneous traffic conditions have originated from the concept of homogeneous behavioural models. Moreover, the identification and incorporation of parameters that determine the complex 2D vehicle behaviour in the behavioural models are among the critical tasks.

This paper considers the existing body of literature on car-following models and their gradual modifications by researchers to include 2D traffic behaviour; followed by an in-depth evaluation of their abilities and weaknesses in modelling the 2D traffic stream. Also included is the review of various simulation frameworks used to model 2D traffic based on these lane-based behavioural models. Thus, the essential questions for this study are as follows:

- (i) How are the car-following models modified to comprehensively simulate 2D traffic by incorporating the lateral and longitudinal behaviour of the traffic?
- (ii) What are the parameters utilised in lane-based models in evaluating the 2D traffic flow?

- (iii) What are different simulation frameworks that have evolved from these car-following models to model mixed (2D) traffic flow?
- (iv) Which parameters are employed in these simulation frameworks to model 2D traffic conditions?

By uncovering these questions, we present the research gaps in the microscopic simulation of mixed or heterogeneous traffic and show directions for future research.

The methodological steps in this review formed its basis on critical analysis and an intensive search of the literature. Numerous articles were critically reviewed to filter the related articles focusing on 2D traffic conditions.

This review is closely associated with several existing studies in the field of microscopic simulation but has its way and edge. Firstly, Mardiaty [6] reviewed the various articles on car-following and lane-changing models for the diverse traffic conditions in Indonesia. The study demonstrated the lack of generic model for all types of traffic conditions. Later, Aghabayk [7] extensively reviewed different car-following models for heterogeneous traffic streams by considering the influence of heavy vehicles on car-following (CF) behaviour and the real-time traffic, thereby suggesting the implementation of various behavioural parameters in the already existing car-following models, especially in terms of the influence of heavy vehicles. Sai Kiran [8] conducted an extensive review of studies on heterogeneous traffic in developing countries, concentrating on the vehicle characteristics, road characteristics, and lateral vehicle behaviour. They suggest the inclusion of lateral clearance and acceleration, along with the establishment of more realistic car-following models for heterogeneous traffic streams. Finally, Mahmud et al. [9] extensively reviewed the evaluation of safety parameters by various commercially-available simulation frameworks, pointing out their strengths and weaknesses and thereby concluding the need for a safety microsimulation model in heterogeneous traffic situations, especially in developing countries. Each of these review articles has a methodological point of view. We concentrate on the parametric aspect of various car-following models and their evolution into accommodating the heterogeneous traffic conditions along with their simulation frameworks.

This review takes place in the following manner: firstly, a brief introduction of 2D models developed from 1D car-following models is discussed, followed by a complete interpretation of each model extended for 2D traffic flow. Secondly, a parametric summary of the car-following models expanded for 2D traffic is discussed. Thirdly, a thorough evaluation of all the simulation frameworks developed from these car-following models to simulate 2D traffic stream is presented, followed by a summary of their shortcomings. Finally, we concluded the findings on the utilisation of 1D car-following models for 2D traffic streams and identified potential gaps in the literature.

To spot relevant studies in these microscopic car-following models, we made use of following academic search engines: Google Scholar, Scopus, IEEE Xplore, ResearchGate, and ScienceDirect. The search word pattern is as follows: “Microscopic,” “Simulation,” “Heterogeneous,” “Mixed traffic,” “car-following,” “No-lane,” “Safety

indicators,” “VISSIM,” “Car-following,” “Traffic shock-waves,” “Driver Modelling,” and “Traffic flow modelling.”

The list of publications considered in this review from the year 1975 to 2019 is shown graphically in Figure 1. Also, the total number of literary works considered for each CF model and their simulation frameworks in this review are outlined in Figure 2.

## 2. Review of Discussion

This chapter includes all the significant contributions made by past researchers in the area of microscopic analysis of traffic flow and safety assessment, and how the concept of heterogeneity has been included and developed from the homogeneous microscopic traffic models has been discussed. The accommodation of vehicle heterogeneity into the car-following models (homogeneous traffic models) is explained in detail below, along with their shortcomings and research gaps, followed by the development of simulation frameworks to include vehicle-heterogeneity based on these car-following models, along with their limitations. Thereby, a detailed summary of vehicle driver behaviour models, as well as simulation frameworks suitable for mixed traffic conditions, is discussed. Findings of this study are outlined at the end of the chapter.

*2.1. Development of Two-Dimensional Microscopic Models from One-Dimensional Models.* Traffic flow behaviour is determined by the response of individual vehicle in different traffic conditions, which can be explained using theoretical models of traffic flow. The single-vehicle dynamics and their interactions are analysed using microscopic simulations, developed based on the analytical and empirical models. Factors such as vehicle lateral displacement, driver behaviour, and influence of surrounding vehicles on driving behaviour determine the interaction between these vehicles [10]. Different vehicle behaviour models have been previously developed, which mostly focused on driving behaviour in car-following conditions (longitudinal interaction only), i.e., in a 1D scenario. A summary of the existing car-following models is presented in this section.

The idea of a car-following model was first conceived by Reuschel (1950) and further developed by scientists in later years [11]. In the models, the cars are expressed as a vector of state variables, considering a particular position ( $x_n$ ), velocity ( $v_n$ ), and acceleration ( $a_n$ ) of the  $n^{\text{th}}$  vehicle that is moving and following a specific path over time ( $t_n$ ).

Figure 3 shows the illustration of the operation of the CF model of  $(n)^{\text{th}}$  car following the  $(n-1)^{\text{th}}$  car. The factors used in the CF model are relative spacing ( $\Delta x = x_{n-1} - x_n$ ) and relative velocity ( $\Delta v = v_{n-1} - v_n$ ) between  $(n)^{\text{th}}$  and  $(n-1)^{\text{th}}$  vehicles. The frequently used CF models are as follows: safe-distance models, cellular automata (CA) models, optimal velocity models, psychophysical models, artificial intelligence models, stimulus-response models, and fuzzy logic models. A vast number of CF models are already presented by many authors [7, 12–15]. The CF models developed for lane-based traffic are discussed below briefly, and Figure 4 exhibits the categorization of various car-following models.



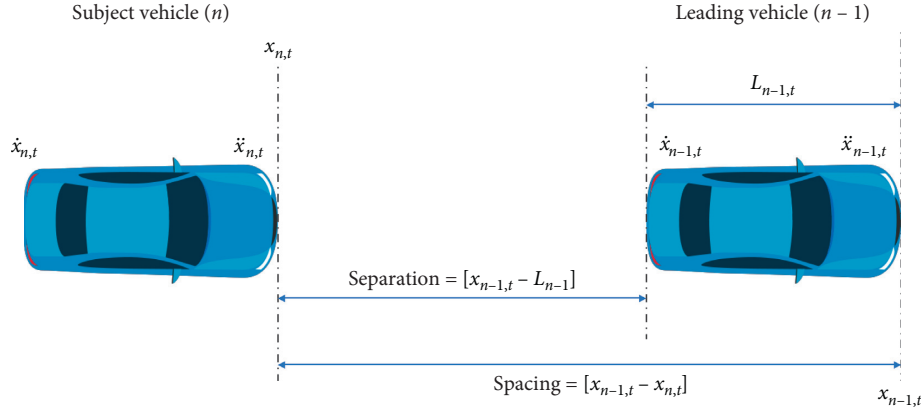


FIGURE 3: Car-following model notations.

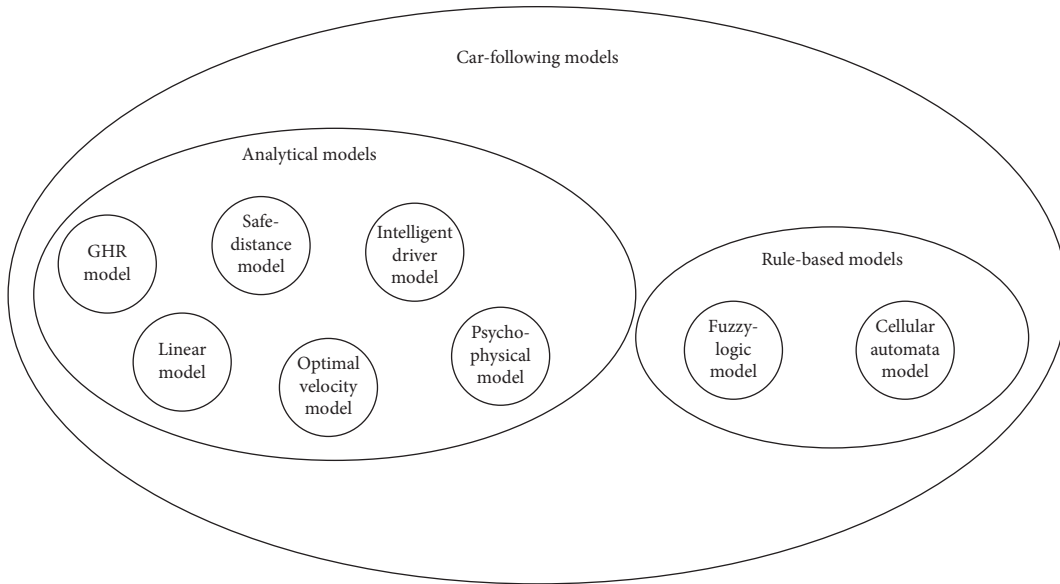


FIGURE 4: Various car-following models for homogeneous traffic conditions.

model is required to extend its application to modelling of multilane traffic; the original model does not take into account the possible risk factors, although they are sometimes considered in time delay and deceleration rate [22]. Currently, there is no report on successful modifications of both linear and IDM models that can allow for evaluation of 2D traffic.

A psychophysical car-following model has been used since 1963 to determine the thresholds and the acceleration of following vehicles [18, 23–27]. Factors commonly considered in car-following models such as emergency braking, free driving, and vehicle following and closing behaviours are incorporated in the model [23]. The CF model has been proven better than other models in mimicking real driving scenarios except that the calibration of its parameters can be challenging. Examples of simulation platforms using psychophysical approach include VISSIM and PARAMICS. However, it is important to note that the 2D traffic modelling using VISSIM has been a complete failure although it is widely used to simulate heterogeneous traffic conditions.

**2.2. GHR Model.** GHR was first introduced in the year 1958 by Chandler [28] to determine the relative velocity between two-lane based vehicles, which is defined as stimulus. It was developed based on the GM model to evaluate vehicle behaviour in car-following and free-flow scenarios [29]. In addition, incorporation of traffic density to determine the stimulus in terms of nonlinearity in the GHR model addresses the limitations in the GM model [30], which expression is as follows:

$$x_{n,t} = \alpha \frac{[x_{n,t} - \xi \Delta t]^\beta}{[x_{n-1,t-\xi \Delta t} - x_{n,t-\xi \Delta t}]^\gamma} [K_{n,t-\xi \Delta t}]^\lambda \cdot [x_{n-1,t-\Delta t} - x_{n,t-\Delta t}]^\rho + \varepsilon_{n,t}, \quad (1)$$

where the term  $\Delta t$  represents the reaction time of the driver and  $x_{n,t}$  is the acceleration/deceleration of the studied vehicle at position  $n$  and time  $t$ . The expression  $t x_{n,t-\Delta t}$  is defined as the velocity of the studied vehicle at position  $n$  and time  $t - \Delta t$ , where  $x_{n-1,t-\Delta t}$  is the speed of the leading vehicle



at the position  $n - 1$  and time  $t - \Delta t$ . The density of traffic is represented by  $[K_{n,t-\xi\Delta t}]$  at the time  $t - \xi\Delta t$ , where the driver sensitivity is a constant  $\alpha$ , the speed is  $\beta$ , the vehicle density is  $\lambda$ , the relative velocity is  $\rho$ , and the spacing is  $\gamma$ . The time lag is expressed as  $\xi\epsilon[0, 1]$ , and the error term associated with the  $(n)^{\text{th}}$  vehicle at time  $t$  is expressed as  $\epsilon_{n,t}$ . The driver's acceleration and deceleration responses in consequence of lane changing behaviour are neglected in this model [31]. Variability in vehicle types is also ignored in the calculation of reaction time using this model [7, 14]. Both GM and GHR models do not properly address the diverse characteristics of different drivers and vehicle types. Other limitations in these stimulus-response models include the absence of acceleration/deceleration limits as well as favourable distances.

Development of the car-following model in diverse traffic conditions using GHR stimulus response was previously reported [32]. Acceleration and deceleration stimuli were applied in two individual models with different vehicle combinations, namely, car-following-truck and truck-following-car in a single lane. In heterogeneous traffic conditions, a single following vehicle affected by several leading vehicles (front, staggered, and right) receives stimuli from multiple sources while the leading vehicle affects only the vehicle next to it. The use of the GM model to simulate the acceleration of the leading vehicle in an approach for identifying the leading vehicle was reported by Choudhury et al. [33]. Amini et al. [34] proposed a case study on car-following-motorcycle behaviour using the GHR model, which compares the longitudinal behaviour of a car following an erratic motorcycle to that of the car-following-car scenario. The finding demonstrated that longer headway is maintained at low speed during the motorcycle-car

interaction compared to the car-car-following scenario. However, the effects of irregular movement of motorcycle within the lane on the behaviour of the following car were not considered in the study.

**2.3. Safe Distance Model.** The car-following model was first developed in the 1950s using equations of motion based on the safe following distance [35]. According to this model, a collision is imminent when uncertainty exists in the leading vehicle operation, which may lead to a reduction in relative spacing between the vehicles, compromising the safe following distance. The research work published by Gipps [36] contributed to a significant development in the safe distance model. The primary factor for the success of this model is its ability to model the actual vehicle behaviour in various traffic conditions. However, the accuracy of the model in estimating the safe distance remains questionable. Moreover, the safe distance model does not take into account the diverse traffic conditions. To incorporate the effects of lateral clearance in the car-following model, Gunay [37] observed the theory of back-end collision in consequence of lateral clearance in the modified Gipps's car-following model, which suggests two factors restricting the top speed of the following vehicle at the end of reaction time. One of the factors is the maximum escape speed (MES), which denotes the speed of the following vehicle during acceleration adjustment at the time when it progresses through the escape passage at its maximum allowable speed. Another factor is the speed that enables the vehicle to have an adequate time to steer laterally ( $t_{\text{veer}}$ ) to prevent a back-end collision. The following are the equations used to model the speed of the subject vehicle that regulates the circumstances:

$$v_n(t + \tau) \leq b_n \tau + \sqrt{(b_n \tau)^2 + 2b_n} \left\{ v_n(t) \frac{\tau}{2} + \frac{\text{MES}^2}{2b_n} + \frac{v_{n-1}^2(t)}{2b_{n-1}} + y_n(t) - y_{n-1}(t) + s_{n-1} \right\}, \quad (2)$$

$$v_n(t + \tau) \leq 2 \frac{y_{n-1}(\text{rest}) - y_n(t) - 0.5\tau v_n(t) - (t_{\text{veer}}/2)\text{MES} - d_{\text{react}}}{t_{\text{veer}} + \tau},$$

where  $b_n$  and  $b_{n-1}$  represent the rate of deceleration of the subject and leading vehicle, respectively,  $d_{\text{react}}$  indicates the distance covered during the reaction time, and  $d_{\text{veer}}$  represents the distance covered during manoeuvring. The speed, flow, and density in a three-lane highway are described in a diagram based on the model that has been validated through prolonged simulation period (60 min), taking into account the traffic conditions during peak and off-peak hours [38]. Similarly, Xu [39] carried out a study on the behaviours of the following vehicle in the car-following model. Gipps's model was modified to include vehicle-type-based parameters for different combinations of cars, trucks, and buses on a single-lane road [40]. The modelling of heterogeneous traffic conditions by modifying Gipps's car-following model was carried out by Lenorzer [41] at a junction in Thane during the evening peak hours. The

modifications were made by considering the reaction time as the shortest reaction time necessary for swerving. The model also examines the influence of lateral clearance on traffic behaviour [37]. The software AIMSUN is used to run this model, which gives results with small deviation (<5%).

**2.4. Optimal Velocity Model.** The optimal velocity model (OVM) was first introduced based on the optimum/desired velocity of the individual following vehicle, which relies upon the relative distance to the leading vehicle; acceleration of the subject vehicle is controlled in a way that the optimum velocity is adjusted according to the trajectory of the leading vehicle [42]. OVM gains considerable attention in recent years due to the use of single-variable function and simple mathematical formulation.

Development of models for heterogeneous traffic conditions using OVM incorporates lateral spacing effects and time-to-collision (TTC) variable. The modifications in the original model enhanced the simplicity in OVM in evaluating the evaluation of traffic jam and stop and go waves. Several CF models incorporate consideration of multiple leading vehicles. It is advisable to use a comprehensive car-following model such as intelligent transport system (ITS) in the investigation of the random vehicles passing the road, which works similarly to a multianticipative car-following model [43–46]. To integrate ITS application with OVM the two-velocity difference model (TVDM) was proposed [47], which is expressed as follows:

$$\ddot{x}_n(t) = a[V((\Delta x)) - \dot{x}_n(t)] + \lambda G(\Delta v_n, \Delta v_{n+1}), \quad (3)$$

where  $G$  denotes a generic function, where  $G(\Delta v_n, \Delta v_{n+1}) = p\Delta v_n + (1-p)\Delta v_{n+1}$ , in which  $p$  is a weighting value that indicates the effect of the leading vehicle on the trajectory of the following vehicle; the value reduces gradually with the decrease in relative distance between following vehicle and leading vehicle. Further changes have reportedly been made in the full-velocity difference (FVD) model [48–50]. The delay in reaction time of the driver is incorporated into the mathematical equation used for development of the FVD model to demonstrate the mechanisms in traffic congestion, as the delay contributes to erratic vehicle behaviour [39, 51].

Jin et al. [52] modified the full-velocity model (VFD) by considering the lateral headway properties between the LV and FV on a single-lane road by proposing a mixed traffic flow-based full-velocity difference CF model. The effects of lateral separation on the LV as a function of the dynamics of FV are formulated below:

$$a_n(t) = \alpha \{V[\Delta x_{n,n+1}(t), \Delta x_{n,n+2}(t)] - v_n(t)\} + kG[\Delta x_{n,n+1}(t), \Delta x_{n,n+2}(t)], \quad (4)$$

where

$$\begin{aligned} V[\Delta x_{n,n+1}(t), \Delta x_{n,n+2}(t)] &= V[(1-p_n)\Delta x_{n,n+1}(t) \\ &\quad + p_n\Delta x_{n,n+2}(t)], \\ G[\Delta x_{n,n+1}(t), \Delta x_{n,n+2}(t)] &= [(1-p_n)\Delta x_{n,n+1}(t) \\ &\quad + p_n\Delta x_{n,n+2}(t)], \\ V[\Delta x] &= 0.5v_{\max}[\tanh(\Delta x - h_c) \\ &\quad + \tanh(h_c)], \end{aligned} \quad (5)$$

where the coefficient of the sensitivity of a driver to the difference between optimal and actual velocities is given by  $\alpha = 1/\tau$  and the response to stimulus  $G(\cdot)$  is represented by sensitivity coefficient  $k = \lambda/\tau^2$ . The lateral clearance is implemented by  $\text{inp}_n = L_{s_n}/L_{s_{\max}}$ , where the lateral clearance is  $L_{s_n}$  between the following vehicle ( $n$ th) and leading vehicle ( $n+1$ ), and the maximum lateral clearance of LV is denoted by  $L_{s_{\max}}$  which does not have any influence on the FV. The notation  $p_n$  denotes that  $n$  follows  $n+2$ , as  $n+1$  is on the other lane.

Modelling of the staggered car-following condition in consideration of TTC variable using OVM allows for the evaluation of the effects of lateral friction on a unidirectional two-lane road [53]. Variations in headway, speed, and acceleration that stimulate the behaviour of the subject vehicle are represented by the visual angle. The modification of the FVD model includes the introduction of the rate of change in visual angles to replace the real-time headway distance and relative speed originally used in the model. Broadening applications of the FVD model allows for consideration of impact of lateral clearance on two-side road in heterogeneous traffic conditions [41, 52]. However, the lateral headway between the leading and subject vehicle is ignored in this model. Yang [54] developed a heterogeneous OVM to analyse the stability of heterogeneous traffic stream consists of car-truck combination. The study demonstrated that the stability of the traffic stream depends heavily on the compositions of the car and truck rather than the density of the traffic. However, the lateral or 2D vehicle behaviour in car-truck combination is not determined in this modified heterogeneous OVM.

Further improvement of the OVM model has been carried out by incorporating the headway distribution and presumed overtaking throughout the escape passageway [55]. The impact of overtaking on the car-following behaviour was the focus in the model. However, the presumed overtaking manoeuvre in the model cannot be guaranteed. A newer model based on the parameters in lateral clearance, and overtaking manoeuvre has been developed and calibrated using the generic algorithm framework. The model allows for extended applications of OVM for heterogeneous traffic stream, which considers the variations in maximum driving speed to determine the optimal driving speed and driver behaviour [56]. It was concluded that the increased anticipation in driving behaviour increases the stability of heterogeneous traffic. Additionally, increase in the lowest maximum speed can result in stop and go waves. The influence of psychological state of the drivers on the headway in heterogeneous traffic conditions was investigated using a model developed from modifications of car-following and FVD models [57]. The finding demonstrated that the driver's psychological state does influence the congestion formation and stop-and-go waves in the traffic. Summary of the models modified to accommodate 2D traffic is presented in Table 2.

**2.5. Cellular Automata Model.** Nagel and Schreckenberg [58] introduced cellular automata (CA) model in the year 1992. Time and space are discrete variables in the model, and the road segment is divided into cells of equal size (typically 7.5 m long), which can hold at least one vehicle or can be empty. The parameters such as acceleration, braking time, and randomisation in types of vehicle are incorporated into the model to demonstrate the longitudinal motion of vehicles. Later, Nagel [59] modified the CA model to be applied on the two-lane traffic. The model considers a few driving rules to reduce the computational workload. Therefore, large-scale modelling of dynamic traffic can be carried out using this model. Model vehicles are required to update



TABLE 2: Parameters used in various simulation frameworks.

Simulation framework	Parameters																			
	Driver parameters										Vehicle parameters									
	$d_{\text{react}}$	$d_{\text{veer}}$	$d_{\text{br}}$	$d_{\text{s}}$	$G_{\text{min}}$	LAD	MES	TEF	$t_{\text{veer}}$	$T_{\text{g}}$	$T_{\text{q}}$	$T_{\text{r}}$	$T_{\text{s}}$	$A$	$D$	$L$	$W$	DS	$\Delta v$	$\Delta d$
AIMSUN	*	*	*	—	—	—	*	—	*	—	—	*	*	*	*	*	*	*	*	*
CARSIM	—	—	—	—	—	—	—	—	—	*	—	*	*	*	*	*	—	*	*	*
CORSIM	*	*	*	—	*	*	—	—	—	—	—	*	*	*	*	*	—	—	*	*
MITSIMLab	—	—	—	—	—	—	—	—	—	*	—	—	—	*	*	*	—	*	*	*
NETSIM	*	—	*	*	—	—	—	—	—	—	—	—	*	—	—	*	—	—	—	*
PARAMICS	—	—	—	—	—	*	—	*	—	*	—	—	*	*	*	—	—	—	*	*
SimTraffic	*	*	*	—	—	—	—	—	—	*	—	*	*	*	*	*	—	—	*	*
SUMO	—	—	*	*	—	—	—	—	—	—	—	*	*	*	*	*	*	*	*	*
VISSIM	—	—	—	—	—	*	—	*	—	*	—	—	*	*	*	*	*	*	*	*

$d_{\text{react}}$  = distance travelled during reaction time;  $d_{\text{veer}}$  = veering distance;  $d_{\text{br}}$  = braking distance;  $d_s$  = distance travelled during simulation time step;  $G_{\text{min}}$  = acceptable minimum GAP; LAD = look ahead distance; MES = maximum escape speed; TEF = threshold value for entering following;  $t_{\text{veer}}$  = veering time;  $T_g$  = time gap;  $T_q$  = queue time;  $T_r$  = reaction time;  $T_s$  = simulation time step;  $A$  = acceleration rate;  $D$  = deceleration rate;  $L$  = vehicle length;  $W$  = vehicle width; DS = desired speed;  $\Delta v$  = relative velocity;  $\Delta d$  = relative distance.

information on the speed, gap requirements, and acceleration/deceleration in multiple cells due to the discretisation of cells of the exact size. Discretisation of cells presents a significant drawback as it risks the loss of information. Representation of all vehicle types cannot take place if a large cell is used, whereas the use of small cell may increase computational workload. In addition, the crucial factor, i.e., variability in headway with regards to vehicle velocity, cannot be evaluated due to the constant cell size.

Lan and Chang [60] described the manoeuvrability and erratic behaviour of motorcycles using the CA model in the year 2003. The CA model used considers the development of a particle-hopping model in the heterogeneous traffic flow composed of only cars and motorcycles. The modifications of the CA model for applications involving other vehicle types has been carried out [61], which considers the flow, speed, and occupancy to accurately determine the collective behaviour of a traffic stream in a spatiotemporal aspect. Later, Lan [62] conducted a similar study by introducing heterogeneous traffic conditions composed of motorcycles using the CA model for a single-lane road. A new concept based on CA simulations was introduced in the later years, which addressed the limited deceleration capability of vehicles [63].

Gundaliya [64] developed a new approach for heterogeneous traffic, by which individual vehicle occupies more than one cell based on its dimension, unlike the traditional CA models, where each vehicle occupies only one cell. Limitation in model validation in respect to real-time traffic data was eliminated due to a reduction in cell size, producing accurate analysis. The cell of the size  $0.9 \text{ m} \times 1.9 \text{ m}$  is used in the model. However, further testing of the model in different traffic conditions was required to broaden its application. Further modifications of the CA model that involved amendment in the acceleration value as a function of vehicle type and velocity to accommodate the heterogeneous traffic conditions have been carried out [65]. Using the refined CA model, an attempt was made to incorporate the erratic motorcycle behaviour in diverse traffic conditions into the model development [63]. However, a comprehensive

approach to include the vehicle size, mechanical properties, lateral arrangement, and lateral gaps between vehicles in heterogeneous traffic conditions must be devised [66] to improve the cell framework in the CA model. The approach was proven useful to model the vehicles that occupy a specific number of traverse positions, but the segregation of these vehicles in various traffic states is not considered [66].

A review by Pandey et al. [67] on several CA models designed for the heterogeneous traffic flow concluded that incorporating the lateral motion in the model as proposed by Mallikarjuna et al. [66] will not result in significant improvement in the model. However, to model a heterogeneous traffic composed of cars and bicycles in urban areas, a new CA model incorporating the variability in lateral movement and headway distribution was considered as reported by Luo et al. [68]. An improved CA model was developed to simulate the congested traffic conditions, adopting cells of 0.5 m in length instead of the traditional cell of 7.5 m in length, to accommodate the actual size of different vehicle types [69]. The model was successfully used in the evaluation on the stability of car-truck combination, where the gap maintenance behaviour between different car-truck combinations is represented in a realistic manner compared to traditional CA models. However, this model does not analyse the lateral distribution of vehicles in the congested traffic, as well as noncongested traffic stream. Pandey et al. [70] proposed a modified CA model that considers preference in lateral position, which facilitates the gradual drifting towards the preferred position on the road. The model also considers the safe gap at the front and back of vehicles and the relationship between area of occupancy, interaction rate, and composition of vehicles. Further exploration on the continuous integration of lateral and longitudinal interactions in the model is required.

**2.6. Fuzzy Logic Model.** Incorporation of fuzzy input variables such as the relative headway distribution, velocity, and acceleration in the fuzzy model was first proposed by Kikuchi et al. [71]. Researchers of later time continue to contribute towards development of fuzzy logic guidelines

based on the car-following models [72–75]. However, significant issues in the models were raised after explication of fuzzy guidelines and calibration of their membership functions. The results generated are impractical unless the drivers' perception in the model is established adequately. Similarly, the heterogeneous traffic flow in car-following behaviour is not incorporated into these models.

The fuzzy logic models determine the lane shifting behaviour of the vehicles. The use of fuzzy logic guidelines to determine the lane adaption behaviour was first reported by Oketch et al. [76]. A similar model to simulate the lane shifting behaviour of the vehicles has also been carried out [77, 78].

*2.7. Other Microscopic Models Developed for Two-Dimensional Traffic Flow.* The models such as car following, lane changing, and gap acceptance are designed specifically for homogeneous traffic conditions and do not have the capacity to reflect the vehicle behaviour in diverse traffic conditions. Compared to homogeneous traffic conditions, poor lane discipline among the drivers during overtaking manoeuvre is often observed in heterogeneous traffic conditions [79]. There have been numerous attempts made in the 1980s to build a conceptual design of vehicle heterogeneity using various simulation models as reported by Palaniswamy et al. [80], which include integration of the car-following model with the lane-changing model. The use of Next Generation Simulation (NGSIM) trajectory data for empirical validation of vehicle-type-dependent car-following model was proposed in Zheng et al. [81], by which the vehicle safety in heterogeneous traffic is determined. Parameters in traffic safety such as time headway (TH), safety margin (SM), and time-to-collision (TTC) are considered to evaluate the safety conditions in the simulations. The results showed that high gap with the leading vehicle is maintained in close vehicle-following scenario when the following vehicle is larger than the leading vehicle. Arasan et al. [82] developed a microsimulation framework called HETEROSIM to conceptualize the heterogeneous traffic conditions. The concept of area of occupancy was introduced to measure traffic concentration in heterogeneous conditions.

Discrete event simulation was designed by incorporating various road geometric conditions and parameters such as shoulder conditions, overtaking speed, and sight distance as observed in Swedish Road Traffic Simulation Model (SWERTS). The simulation considers the different road geometries: single-lane with a two-way traffic of 3.75 m width; an intermediate lane of 5.50 m width; two-lane road of 7.5 m width; and four-lane road. A new approach called potential field model designated for modelling of a two-way traffic stream was reported in the study of Chakroborty et al. [83]. The paper reported on two response models of a given driving simulation: steering response model (SRM) to estimate the veering angle over time and acceleration response model to determine the rate of acceleration/deceleration over time. Obstacles or any potential threats on the road form the primary feature of

this field model. The potential risks associated with the barriers are measured in the construction of potential field function. However, this model was proven unfit to model the over-capacity or congested traffic due to insufficient data on individual driver behaviour under such conditions. Asaithambi et al. [84] evaluated the performance of car-following models such as Gipps, IDM, Krauss Model, Das, and Asundi based on the measure of effectiveness (MoE), which utilises field data obtained from heterogeneous traffic conditions. The report concluded that Gipps and IDM produce realistic values for critical parameters compared to the rest of the models. However, the performance evaluation was limited by the fact that no consideration was given to the integrated behaviour of different vehicle combinations.

There is a method recently developed that takes into account the heterogeneous traffic conditions over a fixed time increment [85] based on the interval scanning technique. Application of the technique can also be extended to model a two-lane traffic condition [86]. Previous approaches in model development for diverse traffic conditions have given too much focus on simulating the vehicle behaviour based on the trajectories of the vehicles. However, evaluation on the impact of individual vehicle is required to model a heterogeneous traffic flow scenario. To assess the complex nature of heterogeneous traffic flow, construction of a comprehensive model is necessary, which takes into consideration the complex individual vehicle interactions, road structures, and the need for model calibration and validation using real-time data. A continuum approach to a car-following model based on Lagrangian coordinates was proposed to model the capacity drop at sag and tunnel bottlenecks in heterogeneous traffic flow [87]. Continuum approach provides an insight into the relationship between stationary speed, acceleration rate, and capacity drop ratio in heterogeneous traffic at sags and tunnel bottlenecks. The influence of heterogeneity in vehicle on car-following behaviour and the associated risk of rear-end crash was evaluated to determine the safety of vehicles in heterogeneous traffic conditions [88]. The desired safety margin (DSM) model was developed based on the sensitivity of the system towards acceleration/deceleration, response time, and vehicle trajectories, which is expressed as coefficients. It was demonstrated that the decrease in DSM and increase in response time will increase the risk of rear-end collision. It was also observed that heterogeneity in driver behaviour and driving style influence the shock wave generation.

A new approach to model heterogeneous traffic flow conditions was recently developed based on a strip-based model [89], utilising Simulation of Mixed Traffic Mobility (SiMTraM), and applied in one of the traditional car-following model framework simulators, SUMO. The method involves the splitting of road segment into small strips to model the continuous lateral displacement rather than the discrete lane changing behaviour. The lateral displacement of vehicles is determined based on a factor called 'benefit', which is measured as the variability between the safe speed surrounding the subject vehicle and current strips. Additionally, the observed data demonstrated that with the

decrease in the width of the pieces, more information can be obtained, thus resulting in better allocation of road space. However, the model has its drawback in a manner that it is unable to understand the two-wheeler behaviour as observed in heterogeneous traffic conditions Lan et al. [90]. A data-driven model was proposed by Papathanasopoulou et al. [91] to integrate comprehensive vehicle behaviour in diverse traffic conditions using the concept of temporary virtual lanes. It was found that incorporation of vehicle type and driver behaviour is critical to model diverse traffic conditions. An experiment to determine the dynamic parameters, i.e., the relation between speed and lateral/longitudinal acceleration in heterogeneous traffic conditions, was conducted by Mahapatra et al. [92]. Different vehicles were analysed to evaluate the dynamic parameters, which showed a linearly proportional relation between lateral and longitudinal acceleration. It was also observed that manoeuvrability is high at low speed due to the increase in longitudinal acceleration as the speed decreases.

**2.8. Area Occupancy Theory.** Area of occupancy theory has been adopted in modelling of heterogeneous traffic conditions to replace the use of fundamental traffic characteristic, namely, density [93]. As heterogeneous traffic stream consists of vehicles having variations in physical and dynamic characteristics, measuring the density based on the number of vehicles is ineffective. Instead, the model must be built based on the evaluation of vehicle composition. The projected area of occupancy takes the different types of vehicles on the road into consideration for model development. Area of occupancy is defined as the observed set of vehicles occupying a given stretch of road within a timeframe. The model is expressed as follows:

$$\text{area occupancy} = \frac{\sum t_i a_i}{TA}, \quad (6)$$

where  $t_i$  is the time spent by vehicle  $i$  on a road stretch within a particular timeframe  $s$  (occupancy time);  $a_i$  is the area of vehicle  $i$  in  $\text{m}^2$ ;  $T$  is the total observation time in  $s$ ; and  $A$  is the area of the entire stretch of a road in  $\text{m}^2$ .

The concept of area of occupancy was developed by Thamizh Arasan et al. [94] to establish an accurate indicator of traffic concentration with diverse traffic flows, by replicating the accurate capacity or level of service on the road in heterogeneous traffic conditions. The idea of vehicle area of occupancy was further developed to indirectly evaluate the density of heterogeneous traffic stream [95, 96]. However, more research is required to understand further about how the occupancy concept will account for differences in dynamic characteristics, such as velocity and acceleration of different vehicles. As the speed of the subject vehicle determines the time variable, the latter, on the other hand, depends on the entrance and exit time of the subject vehicle, imparting doubt on the dependence relation in the model. Moreover, when the area occupancy is measured at an instantaneous time, the time variable can be ignored, which results in negligence of dynamic characteristics of the vehicles. Therefore, in-depth

understanding of the impact of traffic composition on area of occupancy is required.

**2.8.1. Summary of Two-Dimensional Model Development from One-Dimensional Models.** The development of 2D models (longitudinal and lateral movement) from the 1D model (car-following and lane-changing models) has been proven insufficient for modelling of mixed traffic flows. Analytical models such as car-following models have demonstrated flexibility with less computational workload than that of the rule-based models, which involves complex rules in simulating the vehicle dynamics. A broader range of parameters must be determined to develop an efficient 2D behavioural model that can accurately replicate the vehicle behaviour in heterogeneous traffic. This review finds that the currently available models cannot be directly utilised to holistically simulate the heterogeneous traffic unless modified to suit a particular scope of study.

Numerous researches have been conducted to fit vehicle heterogeneity and lateral headway into the currently available car-following models. This is observed in the huge number of reports on extensive modifications conducted on GHR and OVM in the recent years. Consideration of truck-car heterogeneous traffic in the GHR model and identification of the leading vehicle are among the modifications done within the framework. On the contrary, the effect of lateral headway distribution has been incorporated into the OVM model. Limitations in these models include inability to model uninterrupted traffic flow despite successful simulation of the vehicle dynamics in congested and stop-and-go traffic conditions. On the other hand, the development of CA models has been proven efficient in replicating the heterogeneous traffic with simple implementation. However, discretisation in the model may lead to the loss of details, in addition to inaccurate real-time dynamic traffic behaviour that could generate errors in observation of vehicle behaviour in heterogeneous traffic.

This review also finds that collection of field data for microscopic study plays a vital role in evaluating the accuracy of the model. Therefore, selection of the exact influential parameters is crucial for interpretation of the complicated vehicle manoeuvres in heterogeneous traffic, such as irregular lane behaviour, headway distribution, aggressive two wheelers, and tailgating. Identification and understanding of the essential parameters in heterogeneous traffic are thus vital for development of a comprehensive model. Table 3 outlines the modifications of car-following models to simulate vehicle heterogeneity.

## 2.9. Integrated Driver Behavioural Models

**2.9.1. Field Theory Approach.** Human psychology has laid the foundation for social behavioural theory. In fact, social behavioural theory is the basis of field theory introduced by Lewin [99]. Initially developed for societal situations, field theory was described as a method for evaluating casual relationships. This theory considers two forces, namely,

TABLE 3: Outline of car-following models.

Car-following model	Type of vehicles considered	Type of study	Parameters	Simulation framework	Final output	Reference
GHR model	Cars, trucks	A single-lane model consisting of acceleration and deceleration responses	Different pairs of leader-follower C-C, T-C, C-T	—	Estimation of speed, relative speed, space headway parameters	Siuhi and Kaseko [32]
	HMV, LMV, 2W	Determination of leader vehicle: (i) Front vehicle (ii) Staggered right vehicle (iii) Staggered left vehicle	(i) Space headway, overlap in lateral direction, type of vehicle, relative velocity (ii) Nine vehicle pairs for acceleration and deceleration responses	—	Estimation of deceleration rates between different vehicle pairs, acceleration function of all vehicle types	Choudhury and Islam [33]
Safe distance model	Cars	Staggered car-following considering right front leader vehicle	Lateral separation, frictional clearance, veering distance, maximum escape speed (i) Frictional clearance directly depends on escape speed (ii) Frictional clearance decreases with an increase in safe following distance	C++	Speed, flow, density diagrams, space-time trajectories	Gunay [37]; Gunay [38]
		Staggered car-following homogeneous model		Matlab	Space-time diagrams for different FC, CS	Xu [39]
	3W, trucks, buses	Abrupt direction changes in staggered car following	Lateral clearances, maximum lateral speed of FV	AIMSUN simulator	Comparison of vehicle counts	Lenorzer et al. [41]
		Gipps's model of various leader-subject vehicle types	Driver reaction time, acceleration, desired speed of FV, size of leader vehicle	C	Flow-speed diagrams for various vehicle pairs	Ravishankar and Mathew [40]
Optimal velocity model	Cars	Front left and angled car-following	Lateral headway, visual angles	Numerical simulation	Headway, velocity differences, lateral separation parameter, space-time trajectories	Jin et al. [52]; Zheng et al. [81]
		Right-hand staggered car following	Visible angle, gap maintenance angle, time-to-collision		Speed and headway variation with time	Jin et al. [53]
		Angled car-following front, left, and right leaders	Two-sided lateral separation		Distance between cars at different time steps	Li et al. [97]
		Staggered car-following two leaders	Lateral headway, escape corridor		Vehicle gap, velocity variations with time	He et al. [98]
Potential field approach	Mixed traffic	Comprehensive model: (i) Car-following (ii) Two-way traffic (iii) Passing and overtaking	Lane widths, lane markings, influence of road edges, the influence of obstacles	Numerical model	Positions of slow-moving vehicles at different time steps; maximum lateral deviation, speed; speed-time and headway-time trajectories	Chakraborty et al. [83]
Strip-based approach	Mixed traffic	Continuous movement in lateral direction instead of lane-changing	(i) Division of road into tiny strips (ii) Safe speeds on surrounding and current strips	SiMTraM, SUMO	Speed-density relationships and capacities are assessed	Mathew et al. [89]

attractive forces (those that move an object towards its goal) and repulsive forces (those that restrict the movements towards the destination). These forces are imposed by the

surrounding stimuli influencing the agent. This theory evaluates the change in an agent's life sphere depending upon its response to the external stimuli. The



implementation of field theory in describing the driver behaviour has been proposed [83, 100]. In this conceptual framework, each driver has a field or life space surrounding him, and external stimuli exist in each living space or field. Different combination of forces is allied with each stimulus. These forces can be attractive or repulsive in nature, and the cumulative effect of these forces determines the behaviour of the driver in the traffic stream.

**2.9.2. Social Force Approach.** Social force theory was first implemented for studying pedestrian dynamics by Helbing and Molnar in 1995 [101, 102]. Social forces are the forces which drive a pedestrian towards the destination by his/her influencing direction and speed. This theory implements three kinds of forces, namely, driving forces, attractive forces, and repulsive forces. Their mathematical forms are shown in equation (7). Driving forces guide the pedestrian in the required direction towards a specific destination. Attractive forces make a pedestrian moving towards an object. Repulsive forces restrict a pedestrian from colliding with moving vehicles or pedestrians and stationary objects. Furthermore, this theory has been extended to model integrated driver behaviour. A social force model has been developed to evaluate the erratic behaviour of motorcycle drivers [103]:

$$\frac{dv_{\alpha}}{dt} = F_{\alpha}^A + \sum_{\beta} F_{\alpha\beta}^R + \sum_{\beta} F_{\alpha}^B, \quad (7)$$

where  $F_{\alpha}^A$  is the driving force of subject  $\alpha$ ,  $v_{\alpha}$  denotes the actual speed of subject  $\alpha$ ,  $F_{\alpha\beta}^R$  denotes the repulsive force from surrounding vehicle  $\beta$  to subject  $\alpha$ , and  $F_{\alpha}^B$  denotes the repulsive force from the infrastructural boundaries.

Huynh et al. [104] implemented attractive forces along with repulsive and acceleration forces to evaluate the grouping behaviour of motorcycles at a signalised intersection. They simulated the vehicle trajectories using the VISWALK simulation framework via the trial-and-error method for parameter modification. However, VISWALK failed to converge the attractive forces in the proposed model for the grouping of motorcycles. Huang et al. [105] explored the utilisation of the social force model for the simulation of vehicles in two-dimensional space. This model was also able to simulate cars and motorcycles simultaneously, where overtaking/passing, queue forming, and turning behaviour were explored at the intersections. Anvari et al. [106] presented a three-layer microscopic model to simulate shared space users at intersections between pedestrians and vehicles by enforcing equal priority. They simulated the agent's behaviour under the mixed stream along with conflict detection and their resolution. Here, the social force model was modified to include the mixed traffic condition in order to develop feasible trajectories, thereby contributing to the development of new street designs. Babu et al. [107] combined the concepts of the intelligent driver model and social force model. They proposed the concept of perception lines to understand and evaluate the 2D movements of motorcycles. This proposed model was able to replicate the trajectories involving the interaction of motorcycles with other

vehicles (e.g., cars, two wheelers, and three-wheeled motorcycles). Hsu et al. [108] utilised a series of attractive and repulsive forces by implementing a force field interactive conceptual framework to evaluate the integrated driving movement of motorcycles. The parameters are calibrated, and the validation outcome of this model revealed that the mean absolute percentage error in velocity was quite high, i.e., 33.8%.

**2.10. Field Observational Data on Two-Dimensional Traffic Conditions.** A few studies have been conducted on analysing and understanding the real-time vehicular interactions at the microscopic level in the mixed traffic stream. A car-following theory with lateral discomfort was developed by formulating the following vehicle's movement as a function of the off-centre of the leader vehicles [37]. Observations such as the reduction in the following distance with increment in the lateral headway between the leader and the follower vehicle and the dependence of the speed of the following vehicle on the route-width were reported. In fact, the spacing between the leader and the subject vehicles are dependent on the vehicle type in mixed traffic conditions [40, 109]. The collection of data took place by global positioning system- (GPS-) enabled vehicles. The mean velocity of the subject vehicle fluctuates at a close range to the leader vehicle in the observation suggesting the following behaviour. They modified some specific parameters in Gipps's model by incorporating a sensitivity parameter to represent the gap variability between the leader-follower pairs (dependent on vehicle type). However, the influence of the surrounding vehicle on the subject vehicle was not considered. Nagaraj et al. [110] performed an extensive data collection on the gap variability maintaining the vehicle behaviour in order to investigate the lateral and the longitudinal placement of vehicles for mixed traffic stream. The minimum lateral clearances for different vehicle types are specified at 0 kmph and at 60 kmph, which vary linearly as a function of the velocity of subject vehicle.

The relationship between lateral gap and vehicle's speed was explored by determining the minimum and maximum lateral gaps [111]. The gap maintaining behaviour in mixed traffic stream depends on the majority number of vehicle types present [112]. Also, Minh et al. [113] determined the existence of a linear relationship between passing motorcycle and lateral headway. Pal et al. [114] analysed the field data to observe the field gap maintenance of vehicles under various traffic situations. In fact, the field gap maintenance is also influenced by the lateral placement of the subject vehicle. The lateral clearance was determined as 1.5 times of the width of the overtaking vehicle [86]. However, this assumption was not substantiated by any filed data analysis. Kanagaraj et al. [115] observed a unique merging behaviour of vehicles under congested traffic conditions in heterogeneous traffic flow. They proposed two new merging behavioural phenomena of drivers, i.e., group merging and vehicle cover merging. Factors such as waiting time, lateral gap, vehicle type, and road characteristics influence the generated merging behaviour. However, the impact of this



behaviour under different driving regimes, densities, and traffic compositions should be further evaluated. Vlahogianni [116] observed and presented the dynamic interactions of two wheelers while filtering and overtaking on urban roads. The factors influencing the two-wheeler drivers on maintaining the spacing with surrounding vehicles for filtering and passing were analysed. Influential factors such as lane spacing, type of leader vehicle, and relative speed with respect to the surrounding vehicles have been identified. However, a large-scale data calibration and validation should be done under different traffic flow situations in order to get an accurate picture of their filtering and overtaking behaviours.

The definition for a leading vehicle in mixed traffic stream is given as the nearest vehicle from the subject vehicle within 30 m with lateral overlap from the subject vehicle's position observed from the collected real-time data [117]. The extraction of these data took place using the semi-automated image processing software known as Trajectory Extractor [118]. Pal et al. [119] determined the lateral gap maintaining the behaviour of vehicles in the mixed traffic stream. Their observational data suggested that there was a significant variability in the lateral gap even when vehicles travelled at a constant speed during passing/overtaking operations. The speed and size of the subject vehicle as well as the speed of the adjacent vehicles would influence the lateral gap variability only when the subject vehicle's speed is beyond the critical speed (vehicle dependent). Abdul Manan et al. [120] conducted an observational study on the factors leading to the erratic speed behaviour of certain motorcyclists. A new software for motorcycle data collection called MECHROM was developed. Their study observed that 42.2% of the motorcyclists exceeded the speed limit, in which 28.6% exceeded the 85<sup>th</sup> percentile speed. Factors such as driver behaviour, motorcycle characteristics, and surrounding traffic stream had a majority impact on the excessive speeding of motorcyclists.

By utilising trajectory data, Das et al. [121] evaluated the parameters affecting the driver's time headway in staggered car-following situations. They concluded that the lead vehicle dimensions affect the following headways of vehicles. More than 90% of the drivers in the staggered car-following condition maintained a headway of less than 2 s. Also, the increase in centreline separation between different vehicle pairs resulted in lower headways. Munigety [109] determined a vehicle-type dependent lateral movement behaviour in which the vehicle is laterally displaced if the threshold value for lateral displacement equals to its own width. The observed data showed that the highest lateral movement occurred in the category of motorcycle, followed by car and heavy vehicle. The observational data on the number of lateral movements of two-wheelers within the lanes were collected as well. Das et al. [122] proposed a dynamic data collection technique for accurate analysis of staggered car-following behaviour in mixed traffic streams. This new experimental study revealed that there exist a positive and an inverse relationship between the longitudinal gap (LG) and the centreline separation (CS). Vehicles with large CS follow the leading vehicle closely, thus leading to lower longitudinal

gaps. Also, subject vehicle speed increases when CS exceeds 2 m. Robert et al. [123] conducted an experimental study to determine the relationship between velocity and time headway. They concluded that drivers moving with different velocities preferred different time headways, while shorter headways were more critical for slower vehicles. Still, major data collection and analysis is required to accommodate different conditions.

#### 2.10.1. Driver Behavioural Data in Mixed Traffic.

Real-time data play a vital role as microscopic models require accurate data analysis, especially in mixed traffic situations. The presence of various vehicle types characterises the mixed traffic flow, which involves peculiar driving features such filtering, swerving, tailgating and shorter gaps. In mixed traffic stream, drivers usually tailgate to their leader vehicle by aligning themselves to one of its lateral edges. This behaviour reduces safety headway. So, whenever there is a possibility for collision or overtaking instead of decelerating, drivers swerve off. This weaving action of these drivers gives rise to integrated lateral and longitudinal motions [124]. This driving behaviour in mixed traffic stream can be analysed by empirical data collection from real-time traffic data. A very limited data on vehicle trajectory are available in case of mixed traffic due to high cost and difficulty in data collection and its extraction and huge variations in vehicle type, size, and dynamics [125].

The observational data are classified as individual and circumstantial data. The individual data include driver's age, education, state of mind, reaction time, and vehicle characteristics such as type, size, and manoeuvrability. The circumstantial data consist of the spatial-temporal traffic state surrounding the driver [126]. The accuracy in inter-relating these two data and their influence on driver behaviour is not fully understood yet due to the involvement of large number of parameters. This leads to the development of certain assumptions while modelling the behavioural interactions of drivers [36, 77]. Significant factors such as distance or time headway, relative velocity between leader-follower pair, traffic stream speed, and variability in the velocity of leader vehicle used in behavioural car-following models have been summarized by Ranney et al. [126]. However, limited studies have been conducted to assess the influence of these parameters [21]. These traffic characteristics can be studied by performing accurate extraction of vehicle trajectory data.

(1) *Summary.* The past observational field data studies considered only the speed of the subject vehicle in modelling the lateral headways. Also, very few researchers used this observational data to substantiate their results. There is a need for analysing field parameters such as vehicle speed, relative velocity, and longitudinal and lateral gaps between front and rear vehicles along with leading and trailing vehicles on the adjacent path in order to grasp a more accurate understanding of driver behaviour in the mixed traffic stream. Vehicle trajectory data consisting of path taken by vehicles at successive time steps are required to analyse and

interrelate the above field parameters. Hence, the vehicle trajectory data are the primary field observational data for modelling the two-dimensional traffic behaviour.

**2.11. Microscopic Simulation Frameworks.** A microscopic simulation framework utilises different vehicle behaviour models to simulate the real-time traffic stream on a wide scale. The framework can be used to analyse and evaluate the traffic characteristics and the performance of various roadway sections at the microscopic level. The homogeneous traffic stream model is the background of most of these simulation frameworks. Due to the complex nature of vehicles in diverse traffic situations, it is challenging to build a simulation framework in this case. Most of these simulation frameworks consider the lateral shifting of vehicles in a discrete manner. In fact, the modelling and the evaluation results of the variability of lateral headway have been lacking accuracy and precision. The factors affecting the lateral manoeuvrability of the following vehicle could not be identified, as the trajectories of the vehicles vary irregularly without any lane discipline. In this section, a short literature about the existing simulation frameworks developed in representing two-dimensional traffic flow and the associated parameters and limitations are presented.

**2.11.1. Simulation Software Developed from GHR Model.** To evaluate and replicate the real-world traffic on various roadway networks, MITSIM (microscopic traffic simulator) was built by Yang et al. [127]. The simulator incorporates the advanced traffic control systems and the traffic guidance systems. MITSIM considers the logics behind the CF model and the lane-changing model in order to model separate vehicles on a single lane and the manoeuvrability of the vehicles concerning the traffic signals. MITSIM incorporates three basic concepts from the car-following model based on different car-following responses, i.e., (i) free flow, (ii) queueing, and (iii) sudden braking. These are the parts of the GHR car-following approach. The information regarding this model has been put forward by many researchers [128, 129].

**2.11.2. Simulation Software Developed from Safe Distance Model.** Based on the safe distance algorithm of the car-following approach, many simulation frameworks such as CORSIM, NETSIM, SUMO, AIMSUM, CARSIM, and SimTraffic have been developed [36, 130, 131].

AIMSUM simulation framework has proved its capability in the modelling of dynamic vehicle assessment. Various researchers utilised AIMSUM for modelling and calibrating individual vehicle dynamics in urban roadways consisting of homogeneous traffic [132–134]. However, for the mixed traffic scenario, Lenorzer et al. [41] came up with a new model using the AIMSUM simulator. Various authors demonstrated the suitability of microsimulation framework CORSIM in mimicking the real-time traffic interactions and traffic control strategies. One of the models was proposed by Halati et al. [135] by introducing the lane-changing

behaviour into the framework. However, this model fails to produce the variable gap between the vehicles, as all vehicles adopt the same gap acceptance behaviour.

The microsimulator SimTraffic adopts nearly the same parameters as that of the CORSIM framework, i.e., vehicle dimensions, rate of change in velocities, and driver behaviour [136]. The response of driver interactions in mixed traffic flow due to the effects of vehicle type and lateral movement was simulated using SiMTraM [109]. The authors concluded that the following headways and lateral movement duration depend upon the subject and leader vehicle types. Also, the longitudinal movement does not affect the lateral driving behaviour of motorbikes. For the microscopic simulation of urban traffic, CARSIM and NETSIM came into existence [131]. Another famous microscopic simulation framework is SUMO, which is a time-discrete and space-continuous based model used to simulate the mixed transportation in urban areas such as pedestrians and public and private transport [89, 137, 138]. The necessary parameters, limitations, and features of these frameworks are discussed separately in Table 4.

**2.11.3. Simulation Frameworks Based on Psychophysical Models.** Psychophysical models that take into account the drivers' psychological behaviour are incorporated in VISSIM and PARAMICS [23, 36]. The microsimulator PARAMICS is efficient in the simulation of road intersections, merging and diverging traffics, and the roundabouts. The modification of the car-following model in the PARAMICS framework to incorporate acceleration and deceleration rates was performed by Munigety et al. [138] by introducing the threshold parameters into the model. In the same way, VISSIM a microscopic time-based simulation framework is used to simulate the urban traffic flow along with public transit. The methodological framework based on constant monitoring of each vehicle at every time step came to existence. Various real-time situations are encountered in the traffic and transit managements such as road traffic structures, lane structure, and signal controls [23]. VISSIM framework is a combination of psychophysical CF model and lane changing algorithm.

The long-lasting problem in a psychophysical model is the consideration and distribution of parameters related to threshold values. To calibrate the real-world traffic accurately, continuous traffic measurement of different vehicle flow situations is of utmost importance [138, 139]. Various researches have carried out and simulated the mixed traffic flow with the help of VISSIM framework to determine the heterogeneous vehicle behaviour [140–143]. The identification and evaluation of conflicts on the freeway with the help of a modified intelligent driver lane-changing model were developed [143]. Later it has been found out that in the VISSIM model, the slow-moving vehicles in the traffic flow quickly formed bottlenecks, contradicting with the real-time scenario [143]. Also, the simulation of lateral movement within the lane is unsuccessful with VISSIM microsimulator. The calibration of the VISSIM traffic model with various parameters

TABLE 4: Outline of all microscopic simulation frameworks.

Simulation frameworks	Applications	Important characteristics				Limitations
		Mixed traffic	Lane changing	Continuous manoeuvre	Vehicle heterogeneity	
AIMSUM	Dynamic traffic assignment, signal planning, highways, VMS	*	*	*	*	Lateral displacement within a lane cannot be modelled, and slow-moving vehicles form a bottleneck in mixed traffic modelling
CARSIM	Urban traffic flow, congestion conditions, motorways	—	*	—	*	Follows the car-following model while modelling emergency braking neglecting driver's behaviour parameters
CORSIM	Merging sections, incidents, signal designs, VMS	—	*	—	*	Gap acceptance behaviour is fixed, no 3D animation, not well validated, no ramp metering
MITSIM	Intersection control, traffic management, ITS, ramp control, VMS	*	*	—	*	Insufficient parameters, difficulty in calibration, the smoother reaction of leading vehicle causing emergency deceleration
NETSIM	Pedestrian modelling, signal designing, congested flow conditions, urban roads	—	*	—	*	Cannot simulate many vehicles, more simulation time than real-time, only for academic purposes
PARAMICS	Intersections, roundabouts, automated signs, congestion conditions, transit controls	*	*	—	*	Unstable traffic assignment, improper control options, inaccuracy in modelling vehicle guidance
SimTraffic	Signal design, intersections, roundabouts, pedestrians	—	*	—	*	Fixed headway, no ramp metering, unstable saturation flow rates
SUMO	Heterogeneous traffic, freeways, intersections,	—	*	—	*	Longer computational rates, discrete lane changing, lateral displacement within a lane cannot be modelled, bottleneck formation due to low-speed vehicles
VISSIM	Heterogeneous traffic, pedestrians, intersections, signal control, transit operations, ramp metering	*	*	—	*	Lateral displacement within a lane cannot be modelled, bottleneck formation due to low-speed vehicles in mixed traffic flow, no traffic assignment algorithm

under different traffic conditions has been carried out by many researchers [144–147]. A new methodological framework for simulating vehicle-following behaviour for mixed traffic conditions has been calibrated with VISSIM-9.0 [148]. Driving behaviour parameters, along with vehicle trajectories, were calibrated for this mixed traffic conditions.

Ratrout et al. [149] have conducted a general study on various macroscopic and microscopic simulators. The summary and potential limitations concerning the 2D traffic conditions of these two platforms, i.e., PARAMICS and VISSIM, are discussed in Table 4 [150]. From all the abovementioned microsimulation frameworks, only three platforms are successful in simulating the mixed traffic flow scenario (i.e., SUMO, VISSIM, and AIMSUN) based on the time-step frameworks. The lateral displacement behaviour is

widely prevalent and time discrete in the case of mixed-flow conditions, which is absent in the case of adopting the space-continuous car-following and lane-changing longitudinal homogeneous models [124]. A microsimulation model was developed and calibrated by Azam et al. [151] using VISSIM for an urban diamond interchange consisting of heterogeneous traffic flow. The authors concluded that VISSIM could model mixed traffic stream and reflect the actual scenario. However, the driving behaviour and the vehicle positions were not accurately modelled, as the network generation is complicated and time-consuming. Hence, the direct utilisation of these frameworks for evaluating the two-dimensional traffic is not recommended. VISSIM was also successful in replicating the field conditions in modelling platoon discharging at a signalised intersection with a maximum of 10% error [152].

Therefore, a new algorithm should be developed to simulate the mixed traffic scenario. This algorithm should consider parameters such as vehicle dimension and other microscopic details such as interactions among different vehicle types and their dynamic trajectories while filtering through the flow of vehicles. Hence, a detailed understanding of various parameters related to the surrounding road environment on the behaviours of the subject vehicle such as vehicle velocity, relative speed, and spacing can be developed. In short, detailed individual vehicle trajectory data are essential for developing a model that could accurately simulate the vehicle behaviour in mixed traffic flow condition. The significance of those parameters required for characterizing the diverse traffic conditions is summarized in Table 2.

(1) *Summary of Simulation Frameworks.* From the open literature, it seems that there is a good number of microscopic simulation models constructed to model lane-following homogeneous traffic. Nevertheless, they have been proven to be inappropriate for modelling two-dimensional traffic situations. In mixed traffic flow, the lateral interaction is quite apparent. Although there have been modifications in the above models to accommodate vehicle heterogeneity, the models fail to give a clear picture on the real-world scenario. Mathew et al. [89, 153] utilised microsimulator frameworks such as SiMTraM and INSWERTS to understand the vehicle behaviours in mixed traffic conditions. Also, the microsimulator VISSIM models the lateral interaction between vehicles based on the longitudinal time-to-collision. Also, drivers maintain a safe distance, which is proportional to the vehicle velocity. Oketch [76] highlighted that depending on vehicle type and speed, drivers tend to adjust their lateral positions to neighbouring vehicles, thus requiring a lot of computational workloads. This longitudinal time-to-collision is therefore not a suitable parameter while modelling the merging, diverging, and overtaking traffics. Furthermore, these models are not adequately calibrated and validated with the macro- and microparameters to mimic the real-world mixed traffic conditions. Therefore, these models should be further validated with the real-time traffic parameters such as driver's threshold, perception, and reaction times. Hence, a model that is capable to simulate the two-dimensional traffic for mixed flow conditions is desirable.

(2) *Concluding Remarks.* For accurate modelling of the mixed traffic flow scenario using microscopic simulation model, the model should be capable in evaluating the lateral and longitudinal behaviours of different vehicle types simultaneously. All the past research works in microscopic modelling fixed the lateral headway of the vehicle with the leading vehicle and neglected the impact of staggered vehicles on the dynamics of the subject vehicle. In mixed traffic flow, the multiple-leader following approach is common; however, it is not considered in the previous homogeneous models. At the same time, only a few developed models consider vehicle heterogeneity or 2D traffic flow. Also, only a handful of researchers evaluated the vehicle trajectory while overtaking and filtering through mixed or no-lane traffic.

Different vehicles have different radius of influence on the surrounding vehicles due to traffic heterogeneity. These parameters were absent in the previous studies. Therefore, a detailed parametric study on the driver behaviour should be conducted.

Since there are two separate homogeneous models that can be used in evaluating the integrated movements of vehicles for mixed traffic flow, there is a need for integrating the lateral and longitudinal behaviours of vehicles as drivers tend to displace laterally while moving longitudinally. The individual vehicle trajectory due to the influence of other vehicles and objects along with variability in the lateral gap was not described by the previous models especially for diverse traffic conditions. The evaluation of vehicles' lateral displacement is discrete; hence, the evaluation of continuous lateral behaviour of different vehicles along with longitudinal manoeuvring is required. Large-scale data collection of field parameters for the purposes of calibration and validation in microscopic simulation is a challenging task, especially for mixed or no-lane traffic flow. Hence, a thorough field survey is necessary.

### 3. Challenges and Research Directions Ahead

The aim of developing 2D driver behavioural models under mixed traffic conditions is to uncover the actual vehicle interactions in the mixed traffic stream, which is essential in the mitigation of congestion and capacity drop and improvement of intelligent driver operations. The major findings are summarized below, along with possible future research directions:

- (1) **Observational data:** parameters governing the interaction between subject vehicle and other vehicles in the traffic stream such as relative gap, relative velocity, spacing with respect to immediate front, back, left, and right vehicles, and spacing with respect to staggered vehicles are essential for understanding the driver's behaviour. Hence, the vehicle trajectory data containing the time-varying vehicle positions are required in order to understand the driver behaviour in mixed traffic conditions.
- (2) **Data collection:** large-scale data collection is required for calibration and validation purposes. Data collection can be done using video recording and floating-car technique. However, most of the field data collections take place for a short road section (~500 m) [154, 155]. The lateral interaction requires longer road stretches for accurate modelling of driver behaviour. Also, the effects of road structure, road geometry, and surrounding conditions on driver's behaviour cannot be comprehensively studied for shorter road stretches. Hence, data collection should be performed in longer road stretches. Also, data collection through video recording is a viable option due to its ability to capture a wide range of parameters representing the realistic traffic behaviour.



- (3) Driving regimes: mixed traffic stream consists of a wide range of driving regimes such as staggered car-following, tailgating, multiple-leader following, filtering, and swerving. Acceleration models are essentially evolved from the simple car-following model upon incorporating the free-flow acceleration mode and other modes such as responsive and nonresponsive car-following modes and acceleration/deceleration modes. Future research work should focus on the capabilities in simulating the margins and transformations in these regimes, along with the determination of driving behaviour in different regimes. Therefore, for accurate modelling, regime boundaries should be set as random variables.
- (4) Manoeuvring patterns: the manoeuvring pattern of motorcycle in mixed traffic stream is unique and erratic. The overtaking manoeuvre, swerving manoeuvre, tailgating, oblique following, and grouping require further research since no developed model can explicitly simulate all the above characteristics in a single modelling approach. Hence, the unique driving pattern of motorcycle should be accurately evaluated and addressed in the future microscopic simulation model.
- (5) Vehicular interactions: the interactions of vehicles in the mixed traffic stream, especially those between two-wheelers and other vehicle types (e.g., cars, trucks, buses, and light commercial vehicles), are an area that requires further research. The development of a simulation model that can replicate and predict actual driver's manoeuvring pattern can then be proposed.
- (6) Lateral movements: the modelling of lateral movement is often performed in a discreet manner such as that in discrete lane or strip. Past researchers assumed that vehicles tended to move discretely between the strips with constant velocity. In mixed traffic stream, different vehicles exhibit varying dynamic characteristics (e.g., lateral movement). Hence, existing lateral movement models must be extended to incorporate the variability in lateral gaps and movements to account for these speed and manoeuvrability differences. In fact, the vehicle lateral position is dependent on driver behaviour and occasion. For example, fast-moving vehicles (cars) prefer to do a lateral shift to the speed lane on the road offering less friction with other vehicles. At the same time, most of the motorcycles and trucks prefer to use the slow lanes. Hence, behavioural models that can capture the desired lateral preferences of different vehicles in different situations should be developed.
- (7) Staggered vehicles: homogeneous traffic flow models do not consider the effect of staggered vehicles on subject vehicle dynamics due to the car-following assumption. In mixed traffic stream, vehicle passing/filtering and swerving are common phenomena. The effect of lateral gap should not be overlooked while studying the dynamics of staggered vehicles. The influence of staggered vehicles on the dynamics of the subject vehicle should be explored.
- (8) Multiple-leader following: the variation in static and dynamic characteristics of vehicles in mixed traffic flow with the presence of multiple leaders is common. The dynamics of subject vehicle under multiple leaders and different traffic conditions should be explored.
- (9) Car-following models for two-dimensional traffic stream: from the above literature, the 2D traffic behaviour can be incorporated by extending the car-following models to simulate mixed traffic conditions. In fact, as compared to the GHR and psychophysical models, the safe-distance modelling approach is more reliable in simulating the longitudinal movements of different vehicles in the mixed traffic stream since drivers tend to maintain a safe gap with others in order to avoid collision. Also, cellular automata models seem to be appropriate in modelling the lateral interactions or lane-changing behaviour of vehicles as compared to fuzzy-logic models. However, the vehicle-type-dependent behaviour in mixed traffic conditions should be incorporated in CF models in order to apprehend the driving behaviour precisely.
- (10) Integrated driving behaviour: the previous microscopic car-following and lane-changing behaviours simulated the lateral and longitudinal vehicular movements in separate models. However, in mixed traffic conditions, drivers tend to make integrated movements by filtering and swerving while maintaining a safe gap with their leader vehicles. Many researchers have integrated both car-following and lane-changing models into one comprehensive model to address this integrated driving behaviour. However, a single model that integrates all driving behavioural parameters is not available. Hence, models such as social force model and field theory model could be integrated with car-following and lane-changing models in order to unveil the integrated driving behaviour in mixed traffic scenarios.

### List of Symbols

$x_n, x_{n-1,t-\Delta t}$ :	Spatial position
$v_n$ :	Speed
$a_n$ :	Acceleration
$t_n$ :	Time
$\Delta x$ :	Relative spacing
$\Delta v$ :	Relative velocity
$\alpha, \beta$ :	Speed parameters
$\lambda$ :	Density parameter
$\rho, \gamma$ :	Spacing parameters



$t_{\text{veer}}$ :	Lateral veering time
$d_{\text{react}}$ :	Distance travelled during reaction time
$d_{\text{veer}}$ :	Distance travelled during veering manoeuvre
$a_{\text{max}}$ :	Maximum acceleration
$K_{n,t-\xi\Delta t}$ :	Traffic density at time $t - \xi\Delta t$
$\varepsilon_{n,t}$ :	Error term for $n^{\text{th}}$ vehicle at time $t$
$\xi \in [0, 1]$ :	Sensitivity time lag parameter
$b_n$ :	Desired deceleration
$Ls_n$ :	Lateral headway
$Ls_{\text{max}}$ :	Maximum lateral headway
$\alpha$ :	Sensitivity coefficient
$\Delta t/\tau$ :	Driver reaction time
1D:	One dimension
2D:	Two dimension
FV:	Following vehicle
LV:	Leading vehicle
CF:	Car following
CA:	Cellular automata
OV:	Optimal velocity
OVM:	Optimal velocity model
IDM:	Intelligent driver model
ITS:	Intelligent transportation systems
TTC:	Time to collision
MES:	Maximum escape speed
TVDM:	Two-velocity difference model
FVDM:	Full-velocity difference model
SRM:	Steering response model
ARM:	Acceleration response model
SUMO:	Simulation of Urban Mobility
NGSIM:	Next Generation Simulation
GFM:	Generalized force model
SWERTS:	Swedish Road Transport Simulation Model.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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## References

- [1] E. Kopits and M. Cropper, *Traffic Fatalities and Economic Growth*, The World Bank, Washington, DC, USA, 2003.
- [2] R. Wang, J. Gong, Y. Wang, H. Chen, S. Chen, and Q. Wang, "Relationship between gross domestic product and accident in China," *Journal of Safety Studies*, vol. 3, no. 1, pp. 19–26, 2017.
- [3] Economic and Social Commission for Asia and the Pacific, "Road safety in Asia and the Pacific," in *Proceedings of the 24th Meeting of UN Road Safety Collaboration*, Bangkok, March 2017.
- [4] D. Pojani and D. Stead, "Sustainable urban transport in the developing world: beyond megacities," *Sustainability*, vol. 7, no. 6, pp. 7784–7805, 2015.
- [5] J. Hawkins and L. Kattan, "Microsimulating heterogeneous traffic: an implementation of the porous flow approach," *Transportation in Developing Economies*, vol. 4, no. 4, 2018.
- [6] R. Mardiyati, N. Ismail, and A. Farooqi, "Review of microscopic model for traffic flow," *ARPN Journal of Engineering and Applied Sciences*, vol. 9, no. 10, pp. 1794–1800, 2014.
- [7] K. Aghabayk, M. Sarvi, and W. Young, "A state-of-the-art review of car-following models with particular considerations of heavy vehicles," *Transport Reviews*, vol. 35, no. 1, pp. 82–105, 2015.
- [8] S. K. M. and A. Verma, "Review of studies on mixed traffic flow: perspective of developing economies," *Transportation in Developing Economies*, vol. 2, no. 1, 2016.
- [9] S. M. S. Mahmud, L. Ferreira, M. S. Hoque, and A. Tavassoli, "Micro-simulation modelling for traffic safety: a review and potential application to heterogeneous traffic environment," *IATSS Research*, vol. 43, no. 1, pp. 27–36, 2018.
- [10] D. Chowdhury, L. Santen, and A. Schadschneider, "Statistical physics of vehicular traffic and some related systems," *Physics Reports*, vol. 329, no. 4–6, pp. 199–329, 2000.
- [11] L. A. Pipes, "An operational analysis of traffic dynamics," *Journal of Applied Physics*, vol. 24, no. 3, pp. 274–281, 1953.
- [12] M. Brackstone and M. McDonald, "Car-following: a historical review," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 2, no. 4, pp. 181–196, 1999.
- [13] T. Toledo, "Driving behaviour: models and challenges," *Transport Reviews*, vol. 27, no. 1, pp. 65–84, 2007.
- [14] S. Siuhi, *Parametric study of stimulus-response behavior incorporating vehicle heterogeneity in car-following models*, Ph.D. thesis, University of Nevada, Las Vegas, NV, USA, 2009.
- [15] Y. Li and D. Sun, "Microscopic car-following model for the traffic flow: the state of the art," *Journal of Control Theory and Applications*, vol. 10, no. 2, pp. 133–143, 2012.
- [16] G. Mahapatra, A. K. Maurya, and P. Chakroborty, "Parametric study of microscopic two-dimensional traffic flow models: a literature review," *Canadian Journal of Civil Engineering*, vol. 45, no. 11, pp. 909–921, 2018.
- [17] W. Helly, "Simulation of bottlenecks in single-lane traffic flow," in *Proceedings of the Theory of Traffic Flow Symposium*, Warren, MI, USA, December 1959.
- [18] G. A. Bekey, G. O. Burnham, and J. Seo, "Control theoretic models of human drivers in car following," *Human Factors: The Journal of the Human Factors and Ergonomics Society*, vol. 19, no. 4, pp. 399–413, 1977.
- [19] M. Aron, "Car following in an urban networks: simulation and experiments," in *Proceedings of Seminar D, 16th PTR Meeting*, Bath, UK, September 1988.
- [20] J. Xing, "A parameter identification of a car following model," in *Proceedings of Second World Congress on ATT*, Yokohama, Japan, 1995.
- [21] M. Treiber, A. Hennecke, and D. Helbing, "Congested traffic states in empirical observations and microscopic simulations," *Physical Review E*, vol. 62, no. 2, pp. 1805–1824, 2000.
- [22] M. Treiber and A. Kesting, "Traffic flow dynamics," in *Traffic Flow Dynamics*, Springer-Verlag, Berlin, Germany, 2012.
- [23] R. Wiedemann, "Simulation des Strassenverkehrsflusses," *Schriftenr. des Instituts für Verkehrswes. der Univ. Karlsruhe*, Band 8, Karlsruhe, Germany, 1975.
- [24] D. N. Lee, "A theory of visual control of braking based on information about time to collision," *Perception*, vol. 5, no. 4, pp. 437–459, 1976.
- [25] U. Wiedemann and R. Reiter, "Microscopic traffic simulation: the simulation system MISSION, background and

- actual state,” CEC Proj, ICARUS (V1052), Final Report, 1992.
- [26] U. Reiter, “Empirical studies as basis for traffic flow models,” in *Proceedings of the Second International Symposium on Highway Capacity*, Sydney, Australia, August 1994.
  - [27] P. Krauss, S. Nagel, and K. Wagner, “The mechanism of flow breakdown in traffic flow models,” in *Proceedings of the 14th International Symposium of Transportation and Traffic Theory*, Jerusalem, Israel, July 1999.
  - [28] R. E. Chandler, R. Herman, and E. W. Montroll, “Traffic dynamics: studies in car following,” *Operations Research*, vol. 6, no. 2, pp. 165–184, 1958.
  - [29] H. Subramanian, *Estimation of Car-Following Models*, Archives Massachusetts Institute of Technology, Cambridge, MA, USA, 1996.
  - [30] K. I. Ahmed, *Modeling drivers’ acceleration and lane changing behavior*, Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 1999.
  - [31] T. Toledo, *The Action-Point Model of the Driver-Vehicle-System*, The Ohio State University, Columbus, OH, USA, 2003.
  - [32] S. Siuhi and M. Kaseko, “Incorporating vehicle mix in stimulus-response car-following models,” *Journal of Traffic and Transportation Engineering (English Edition)*, vol. 3, no. 3, pp. 226–235, 2016.
  - [33] C. F. Choudhury and M. M. Islam, “Modelling acceleration decisions in traffic streams with weak lane discipline: a latent leader approach,” *Transportation Research Part C: Emerging Technologies*, vol. 67, pp. 214–226, 2016.
  - [34] E. Amini, M. Tabibi, E. R. Khansari, and M. Abhari, “A vehicle type-based approach to model car following behaviors in simulation programs (case study: car-motorcycle following behavior),” *IATSS Research*, vol. 43, no. 1, pp. 14–20, 2018.
  - [35] C. L. Azevedo, J. L. Cardoso, and M. E. Ben-Akiva, “Probabilistic safety analysis using traffic microscopic simulation,” 2018, <http://arxiv.org/abs/1810.04776>.
  - [36] P. G. Gipps, “A behavioural car following model for computer simulation,” *Transportation Research Part B: Methodological*, vol. 15, no. 2, pp. 105–111, 1981.
  - [37] B. Gunay, “Car following theory with lateral discomfort,” *Transportation Research Part B: Methodological*, vol. 41, no. 7, pp. 722–735, 2007.
  - [38] B. Gunay, “Rationality of a non-lane-based car-following theory,” *Proceedings of the Institution of Civil Engineers - Transport*, vol. 162, no. 1, pp. 27–37, 2009.
  - [39] R. G. Xu, “Multiple traffic jams in full velocity difference model with reaction-time delay,” *International Journal of Simulation Modelling*, vol. 14, no. 2, pp. 325–334, 2015.
  - [40] K. V. R. Ravishankar and T. V. Mathew, “Vehicle-type dependent car-following model for heterogeneous traffic conditions,” *Journal of Transportation Engineering*, vol. 137, no. 11, 2011.
  - [41] A. Lenorzer, J. Casas, R. Dinesh et al., “Modelling and simulation of mixed traffic,” in *Proceedings of the Australasian Transport Research Forum 2015*, pp. 1–12, Sydney, Australia, September–October 2015.
  - [42] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, and Y. Sugiyama, “Dynamical model of traffic congestion and numerical simulation,” *Physical Review E*, vol. 51, no. 2, pp. 1035–1042, 1995.
  - [43] T. Nagatani, “Stabilization and enhancement of traffic flow by the next-nearest-neighbor interaction,” *Physical Review E*, vol. 60, no. 6, pp. 6395–6401, 1999.
  - [44] K. Hasebe, A. Nakayama, and Y. Sugiyama, “Dynamical model of a cooperative driving system for freeway traffic,” *Physical Review E*, vol. 68, no. 2, 2003.
  - [45] H. X. Ge, S. Q. Dai, L. Y. Dong, and Y. Xue, “Stabilization effect of traffic flow in an extended car-following model based on an intelligent transportation system application,” *Physical Review E*, vol. 70, no. 6, 2004.
  - [46] Z. Li and R. Zhang, “An extended non-lane-based optimal velocity model with dynamic collaboration,” *Mathematical Problems in Engineering*, vol. 2013, Article ID 124908, 8 pages, 2013.
  - [47] H. X. Ge, R. J. Cheng, and Z. P. Li, “Two velocity difference model for a car following theory,” *Physica A: Statistical Mechanics and Its Applications*, vol. 387, no. 21, pp. 5239–5245, 2008.
  - [48] Y. Li, D. Sun, W. Liu et al., “Modeling and simulation for microscopic traffic flow based on multiple headway, velocity and acceleration difference,” *Nonlinear Dynamics*, vol. 66, no. 1-2, pp. 15–28, 2011.
  - [49] X. H. Cai, G. H. Peng, C. Q. Liu, B. F. Cao, and M. X. Tuo, “Optimal velocity difference model for a car-following theory,” *Physics Letters A*, vol. 375, no. 45, pp. 3973–3977, 2011.
  - [50] L. Jing, P. Juan, and Z. Gao, “Improvement of the full velocity difference car-following model,” *Journal of Computer Information Systems*, vol. 7, no. 10, 2011.
  - [51] J. Zhou, Z.-K. Shi, and J.-L. Cao, “Nonlinear analysis of the optimal velocity difference model with reaction-time delay,” *Physica A: Statistical Mechanics and Its Applications*, vol. 396, pp. 77–87, 2014.
  - [52] S. Jin, D. Wang, P. Tao, and P. Li, “Non-lane-based full velocity difference car following model,” *Physica A: Statistical Mechanics and Its Applications*, vol. 389, no. 21, pp. 4654–4662, 2010.
  - [53] S. Jin, D.-H. Wang, C. Xu, and Z.-Y. Huang, “Staggered car-following induced by lateral separation effects in traffic flow,” *Physics Letters A*, vol. 376, no. 3, pp. 153–157, 2012.
  - [54] D. Yang, P. Jin, Y. Pu, and B. Ran, “Stability analysis of the mixed traffic flow of cars and trucks using heterogeneous optimal velocity car-following model,” *Physica A: Statistical Mechanics and its Applications*, vol. 395, pp. 371–383, 2014.
  - [55] Z. C. He, Y. M. Wang, W. B. Sun, P. Y. Huang, L. C. Zhang, and R. X. Zhong, “Modelling car-following behaviour with lateral separation and overtaking expectation,” *Transportmetrica B: Transport Dynamics*, vol. 4, no. 3, pp. 223–239, 2016.
  - [56] F. Sun, J. Wang, R. Cheng, and H. Ge, “An extended heterogeneous car-following model accounting for anticipation driving behavior and mixed maximum speeds,” *Physics Letters A*, vol. 382, no. 7, pp. 489–498, 2018.
  - [57] J. Wang, F. Sun, R. Cheng, and H. Ge, “An extended heterogeneous car-following model with the consideration of the drivers’ different psychological headways,” *Physica A: Statistical Mechanics and Its Applications*, vol. 506, pp. 1113–1125, 2018.
  - [58] K. Nagel, M. Schreckenberg, K. Nagel, and M. Schreckenberg, “A cellular automaton model for freeway traffic,” *Journal de Physique I*, vol. 2, no. 12, pp. 2221–2229, 1992.
  - [59] K. Nagel, D. E. Wolf, P. Wagner, and P. Simon, “Two-lane traffic rules for cellular automata: a systematic approach,” *Physical Review E*, vol. 58, no. 2, pp. 1425–1437, 1998.
  - [60] L. W. Lan and C.-W. Chang, “Motorbike’s moving behavior in mixed traffic: particle-hopping model with cellular

- automata," *Journal of the Eastern Asia Society for Transportation Studies*, vol. 5, pp. 23–37, 2003.
- [61] L. Lan and C. Hsu, "Using cellular automata to explore spatiotemporal traffic patterns," in *Proceedings of the International Conference of Hong Kong Society*, pp. 1–10, Hong Kong, December 2005.
  - [62] J.-p. Meng, S.-q. Dai, L.-y. Dong, and J.-f. Zhang, "Cellular automaton model for mixed traffic flow with motorcycles," *Physica A: Statistical Mechanics and Its Applications*, vol. 380, no. 1–2, pp. 470–480, 2007.
  - [63] L. W. Lan, Y.-C. Chiou, Z.-S. Lin, and C.-C. Hsu, "A refined cellular automaton model to rectify impractical vehicular movement behavior," *Physica A: Statistical Mechanics and Its Applications*, vol. 388, no. 18, pp. 3917–3930, 2009.
  - [64] P. J. Gundaliya, T. V. Mathew, and S. L. Dhingra, "Heterogeneous traffic flow modelling for an arterial using grid based approach," *Journal of Advanced Transportation*, vol. 42, no. 4, pp. 467–491, 2007.
  - [65] C. Mallikarjuna and K. R. Rao, "Cellular Automata model for heterogeneous traffic," *Journal of Advanced Transportation*, vol. 43, no. 3, pp. 321–345, 2009.
  - [66] C. Mallikarjuna and K. R. Rao, "Heterogeneous traffic flow modelling: a complete methodology," *Transportmetrica*, vol. 7, no. 5, pp. 321–345, 2011.
  - [67] G. Pandey, K. R. Rao, and D. Mohan, *Traffic and Granular Flow'01*, Springer Science & Business Media, Berlin, Germany, 2013.
  - [68] Y. Luo, B. Jia, J. Liu, W. H. K. Lam, X. Li, and Z. Gao, "Modeling the interactions between car and bicycle in heterogeneous traffic," *Journal of Advanced Transportation*, vol. 49, no. 1, pp. 29–47, 2015.
  - [69] D. Kong, G. F. List, X. Guo, and D. Wu, "Modeling vehicle car-following behavior in congested traffic conditions based on different vehicle combinations," *Transportation Letters*, vol. 10, no. 5, 2016.
  - [70] G. Pandey, K. R. Rao, and D. Mohan, "Modelling vehicular interactions for heterogeneous traffic flow using cellular automata with position preference," *Journal of Modern Transportation*, vol. 25, pp. 163–177, 2017.
  - [71] S. Kikuchi and P. Chakroborty, "Car following model based on fuzzy inference system," *Transportation Research Record*, vol. 1365, 1992.
  - [72] M. McDonald, J. Wu, and M. Brackstone, "Development of a fuzzy logic based microscopic motorway simulation model," in *Proceedings of Conference on Intelligent Transportation Systems*, Boston, MA, USA, November 2002.
  - [73] P. Chakroborty and S. Kikuchi, "Evaluation of the General Motors based car-following models and a proposed fuzzy inference model," *Transportation Research Part C: Emerging Technologies*, vol. 7, no. 4, pp. 209–235, 1999.
  - [74] P. Zheng and M. McDonald, "Application of fuzzy systems in the car-following behaviour analysis," in *Fuzzy Systems and Knowledge Discovery. FSKD 2005, Lecture Notes in Computer Science*, vol. 3613, pp. 782–791, Springer, Berlin, Germany, 2006.
  - [75] Q. Gao, S. Hu, and C. Dong, "The modeling and simulation of the car-following behavior based on fuzzy inference," in *Proceedings of the 2008 International Workshop on Modeling, Simulation and Optimization WMSO 2008*, pp. 322–325, Hong Kong, China, December 2009.
  - [76] T. Oketch, "New modeling approach for mixed-traffic streams with nonmotorized vehicles," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1705, no. 1, pp. 61–69, 2000.
  - [77] P. Hidas, "Modelling lane changing and merging in microscopic traffic simulation," *Transportation Research Part C-emerging Technologies*, vol. 10, no. 5-6, pp. 351–371, 2002.
  - [78] H. Wei, J. Lee, Q. Li, and C. J. Li, "Observation-based lane-vehicle assignment hierarchy: microscopic simulation on urban street network," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1710, no. 1, pp. 96–103, 2000.
  - [79] S. K. Mayakuntla and A. Verma, "Cell transmission modeling of heterogeneous disordered traffic," *Journal of Transportation Engineering, Part A: Systems*, vol. 145, no. 7, Article ID 4019027, 2019.
  - [80] S. P. Palaniswamy, G. Gynnerstedt, and Y. R. Phull, "Indo-Swedish road traffic simulation model: generalized traffic system simulator," in *Transportation Research Record*, pp. 72–80, 1985.
  - [81] L. Zheng, C. Zhu, Z. He, T. He, and S. Liu, "Empirical validation of vehicle type-dependent car-following heterogeneity from micro- and macro-viewpoints," *Transportmetrica B: Transport Dynamics*, vol. 7, no. 1, 2019.
  - [82] V. T. Arasan and G. Dhivya, "Simulation of highly heterogeneous traffic flow characteristics," in *Proceedings of the 24th European Conference on Modelling and Simulation, ECMS 2010*, Kuala Lumpur, Malaysia, June 2010.
  - [83] P. Chakroborty, S. Agrawal, and K. Vasishtha, "Microscopic modeling of driver behavior in uninterrupted traffic flow," *Journal of Transportation Engineering*, vol. 130, no. 4, pp. 438–451, 2004.
  - [84] G. Asaithambi, V. Kanagaraj, K. K. Srinivasan, and R. Sivanandan, "Study of traffic flow characteristics using different vehicle-following models under mixed traffic conditions," *Transportation Letters*, vol. 10, no. 2, pp. 92–103, 2016.
  - [85] V. T. Arasan and R. Z. Koshy, "Methodology for modeling highly heterogeneous traffic flow," *Journal of Transportation Engineering*, vol. 131, no. 7, pp. 544–551, 2005.
  - [86] P. P. Dey, S. Chandra, and S. Gangopadhyay, "Simulation of mixed traffic flow on two-lane roads," *Journal of Transportation Engineering*, vol. 134, no. 9, 2008.
  - [87] K. Wada, I. Martínez, and W.-L. Jin, "Continuum car-following model of capacity drop at sag and tunnel bottlenecks," *Transportation Research Part C: Emerging Technologies*, pp. 1–17, 2019.
  - [88] J. Zhang, Y. Wang, and G. Lu, "Impact of heterogeneity of car-following behavior on rear-end crash risk," *Accident Analysis and Prevention*, vol. 125, pp. 275–289, 2019.
  - [89] T. V. Mathew, C. R. Munigety, and A. Bajpai, "Strip-based approach for the simulation of mixed traffic conditions," *Journal of Computing in Civil Engineering*, vol. 29, no. 5, 2013.
  - [90] L. W. Lan, Y. C. Chiou, Z. S. Lin, and C. C. Hsu, "Cellular automaton simulations for mixed traffic with erratic motorcycles' behaviours," *Physica A: Statistical Mechanics and its Applications*, vol. 389, no. 10, pp. 2077–2089, 2010.
  - [91] V. Papathanasopoulou and C. Antoniou, "Towards an integrated longitudinal and lateral movement data-driven model for mixed traffic," *Transportation Research Procedia*, vol. 37, pp. 489–496, 2019.
  - [92] G. Mahapatra and A. K. Maurya, "Dynamic parameters of vehicles under heterogeneous traffic stream with non-lane discipline: an experimental study," *Journal of Traffic and Transportation Engineering*, vol. 5, no. 5, pp. 386–405, 2018.



- [93] C. Mallikarjuna and K. R. Rao, "Area occupancy characteristics of heterogeneous traffic," *Transportmetrica*, vol. 2, no. 3, pp. 223–236, 2006.
- [94] V. Thamizh Arasan and G. Dhivya, "Measuring heterogeneous traffic density," *World Academy of Science, Engineering and Technology*, vol. 46, no. 10, pp. 342–346, 2008.
- [95] S. I. Khan and P. Maini, "Modeling heterogeneous traffic flow," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1678, no. 1, pp. 234–241, 1999.
- [96] R. Nair, H. S. Mahmassani, and E. Miller-Hooks, "A porous flow approach to modeling heterogeneous traffic in disordered systems," *Transportation Research Part B: Methodological*, vol. 45, no. 9, pp. 1331–1345, 2011.
- [97] Y. Li, L. Zhang, S. Peeta et al., "Non-lane-disciplined-based car-following model considering the effect of two-sided lateral gaps," *Journal of Non-Linear Dynamics*, vol. 80, no. 1, pp. 227–238, 2015.
- [98] Z. C. He, Y. M. Wang, W. B. Sun, P. Y. Huang, L. C. Zhang, and R. X. Zhong, "Modelling car-following behaviour with lateral separation and overtaking expectation," *Transportmetrica B: Transport Dynamics*, vol. 4, no. 3, pp. 223–239, 2015.
- [99] K. Lewin, F. Heider, and G. M. Heider, *Principles of Topological Psychology*, McGraw-Hill Publications in Psychology, New York, NY, USA, 1936.
- [100] A. L. Berthoume, M. R. E. Romoser, J. Collura, and D. Ni, "Towards a social psychology-based microscopic model of driver behavior and decision-making: modifying Lewin's field theory," *Procedia Computer Science*, vol. 32, pp. 816–821, 2014.
- [101] D. Helbing and P. Molnár, "Social force model for pedestrian dynamics," *Physical Review E*, vol. 51, no. 5, pp. 4282–4286, 1995.
- [102] D. Helbing and B. Tilch, "Generalized force model of traffic dynamics," *Physical Review E*, vol. 58, no. 1, pp. 133–138, 1998.
- [103] L. Xuan, N. Graduate, and S. Hanaoka, "An application of social force approach for motorcycle dynamics," in *Proceedings of the 9th International Conference of Eastern Asia Society for Transportation Studies*, Jeju, South Korea, May 2011.
- [104] D. N. Huynh, M. Boltze, and A. T. Vu, "Modelling mixed traffic flow at signalized intersection using social force model," *Journal of the Eastern Asia Society for Transportation Studies*, vol. 10, pp. 1734–1749, 2013.
- [105] W. Huang and M. Fellendorf, "Social force model for vehicle simulation on operational level," in *Proceedings of the 19th Intelligent Transport Systems World Congress, ITS 2012*, Wien, Austria, October 2012.
- [106] B. Anvari, M. G. H. Bell, A. Sivakumar, and W. Y. Ochieng, "Modelling shared space users via rule-based social force model," *Transportation Research Part C: Emerging Technologies*, vol. 51, pp. 83–103, 2015.
- [107] F. A. M. Babu, P. Vortisch, and T. V. Mathew, "Modelling of motorcycle movements in mixed traffic conditions," in *Proceedings of the Transportation Research Board (TRB) 94th Annual Meeting*, Washington, DC, USA, January 2015.
- [108] T. Hsu and Y. Chen, "Mobility force model for microscopic simulation of motorcycle traffic behavior," in *Proceedings of the 2015 International Conference on Modeling, Simulation and Applied Mathematics*, Phuket, Thailand, August 2015.
- [109] C. R. Munigety, "Modelling behavioural interactions of drivers' in mixed traffic conditions," *Journal of Traffic and Transportation Engineering*, vol. 5, no. 4, pp. 284–295, 2018.
- [110] P. K. Nagaraj, B. N. George, and K. J. John, "A study of linear and lateral placement of vehicles in mixed traffic environment through video-recording," *Highway Research Bulliten, Indian Road Congress*, vol. 42, pp. 105–136, 1990.
- [111] B. Singh, *Simulation and Animation of Heterogeneous Traffic on Urban Roads*, PhD. thesis, IIT Kanpur, Kanpur, India, 1999.
- [112] C. Mallikarjuna, *Analysis and modelling of heterogeneous traffic*, Ph.D. thesis, IIT Delhi, New Delhi, India, 2007.
- [113] C. C. Minh, S. Matsumoto, and K. Sano, "Characteristics of passing and paired riding maneuvers of motorcycle," *Journal of the Eastern Asia Society for Transportation Studies*, vol. 6, pp. 186–197, 2005.
- [114] D. Pal and C. Mallikarjuna, "Cellular automata cell structure for modeling heterogeneous traffic," *European Transport*, vol. 45, pp. 50–63, 2010.
- [115] V. Kanagaraj, K. K. Srinivasan, R. Sivanandan, and G. Asaithambi, "Study of unique merging behavior under mixed traffic conditions," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 29, pp. 98–112, 2015.
- [116] E. I. Vlahogianni, "Powered-Two-Wheelers kinematic characteristics and interactions during filtering and overtaking in urban arterials," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 24, pp. 133–145, 2014.
- [117] V. Kanagaraj, G. Asaithambi, T. Toledo, and T. C. Lee, "Trajectory data and flow characteristics of mixed traffic," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2491, no. 1, pp. 1–11, 2015.
- [118] T. Lee, J. W. Polak, and M. G. H. Bell, *Trajectory Extractor Version 1.0*, Centre for Transport Studies, Imperial College, London, UK, 2008.
- [119] D. Pal and M. Chunchu, "Modeling of lateral gap maintaining behavior of vehicles in heterogeneous traffic stream," *Transportation Letters*, vol. 11, no. 7, pp. 373–381, 2017.
- [120] M. M. Abdul Manan, J. S. Ho, S. T. M. Syed Tajul Arif, M. R. Abdul Ghani, and A. Várhelyi, "Factors associated with motorcyclists' speed behaviour on Malaysian roads," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 50, pp. 109–127, 2017.
- [121] S. Das, A. K. Maurya, and A. K. Budhkar, "Determinants of time headway in staggered car-following conditions," *Transportation Letters*, vol. 11, no. 8, pp. 447–457, 2017.
- [122] S. Das, A. K. Maurya, and A. K. Budhkar, "Dynamic data collection of staggered-following behavior in non-lane-based traffic streams," *SN Applied Sciences*, vol. 1, no. 6, pp. 2–8, 2019.
- [123] R. Tscharn, F. Naujoks, and A. Neukum, "The perceived criticality of different time headways is depending on velocity," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 58, pp. 1043–1052, 2018.
- [124] C. R. Munigety and T. V. Mathew, "Towards behavioral modeling of drivers in mixed traffic conditions," *Transportation in Developing Economies*, vol. 2, no. 1, 2016.
- [125] G. Asaithambi, V. Kanagaraj, and T. Toledo, "Driving behaviors: models and challenges for non-lane based mixed traffic," *Transportation in Developing Economies*, vol. 2, no. 2, 2016.
- [126] T. A. Ranney, "Psychological factors that influence car-following and car-following model development," *Transportation Research Part F: Traffic Psychology and Behaviour*, vol. 2, no. 4, pp. 213–219, 1999.

- [127] Q. Yang and H. N. Koutsopoulos, "A microscopic traffic simulator for evaluation of dynamic traffic management systems," *Transportation Research Part C: Emerging Technologies*, vol. 4, no. 3, pp. 113–129, 1996.
- [128] Q. Yang, H. N. Koutsopoulos, and M. E. Ben-Akiva, "Simulation laboratory for evaluating dynamic traffic management systems," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1710, no. 1, pp. 122–130, 2000.
- [129] R. Balakrishna, C. Antoniou, M. Ben-Akiva, H. N. Koutsopoulos, and Y. Wen, "Calibration of microscopic traffic simulation models: methods and application," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1999, no. 1, pp. 198–207, 2007.
- [130] P. G. Gipps, "A model for the structure of lane-changing decisions," *Transportation Research Part B: Methodological*, vol. 20, no. 5, pp. 403–414, 1986.
- [131] R. F. Benekohal, "Procedure for validation of microscopic traffic flow simulation models," *Transportation Research Record*, vol. 5, pp. 190–202, 1991.
- [132] S. Agrawal, H. Zheng, S. Peeta, and A. Kumar, "Routing aspects of electric vehicle drivers and their effects on network performance," *Transportation Research Part D: Transport and Environment*, vol. 46, pp. 246–266, 2016.
- [133] J. Barceló, E. Codina, J. Casas, J. L. Ferrer, and D. García, "Microscopic traffic simulation: a tool for the design, analysis and evaluation of intelligent transport systems," *Journal of Intelligent and Robotic Systems: Theory and Applications*, vol. 41, no. 2-3, pp. 173–203, 2005.
- [134] R. a Rajasakran, "Aimsun micro-simulation—a practical application: micro-simulation of the N1 freeway," in *Proceedings of the 27th Southern African Transport Conference*, Pretoria, South Africa, July 2008.
- [135] A. Halati, H. Lieu, and S. Walker, "CORSIM—corridor traffic simulation model," in *Proceedings of the Traffic Safety in the 21st Century: Challenges, Innovations and Opportunities*, Chicago, IL, USA, June 1997.
- [136] S. A. T. Jeff and G. Gerken, "Analysis of traffic impacts at isolated light rail transit (LRT) crossings using SimTraffic," in *Proceedings of the Improving Transportation Systems Safety and Performance, Spring Conference and Exhibit*, p. 27, Monterey, CA, USA, March 2001.
- [137] S. Krauss, *Microscopic Modelling of Traffic Flow: Investigation of Collision Free Vehicle Dynamics*, Universität zu Köln: Mathematisches Institut, Köln, Germany, 1998.
- [138] C. R. Munigety and T. V. Mathew, "Towards behavioral modeling of drivers in mixed traffic conditions," *Transportation in Developing Economies*, vol. 2, no. 1, pp. 1–20, 2016.
- [139] S. A. Boxill and L. Yu, *An Evaluation of Traffic Simulation Models for Supporting ITS Development*, Center for Transportation Training and Research, Texas Southern University, Houston, TX, USA, 2000.
- [140] P. Raj and V. Khode, "Forecasting of traffic simulation model under heterogeneous traffic condition," *International Research Journal of Engineering and Technology*, vol. 3, no. 4, pp. 2395–2396, 2016.
- [141] S. Kaur, *Use of Vissim in Heterogeneous Traffic Modelling—A Review Lecture in Civil Engineering*, Government Polytechnic, Ahmedabad, India, 2015.
- [142] P. Manjunatha, P. Vortisch, T. T. V. Mathew, and M. Manjunatha, "Methodology for the calibration of VISSIM in mixed traffic," in *Proceedings of the Transportation Research Board (TRB) 92nd Annual Meeting*, Washington, DC, USA, January 2013.
- [143] H. Park, C. Bhamidipati, and B. Smith, "Development and evaluation of enhanced IntelliDrive-enabled lane changing advisory algorithm to address freeway merge conflict," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2243, no. 1, pp. 146–157, 2011.
- [144] T. Oketch and M. Carrick, "Calibration and validation of a micro-simulation model in network analysis," in *Proceedings of the 84th TRB Annual Meeting*, Washington, DC, USA, January 2005.
- [145] B. (Brian) Park, J. Won, and I. Yun, "Application of microscopic simulation model calibration and validation procedure: case study of coordinated actuated signal system," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1978, pp. 113–122, 2006.
- [146] G. Gomes, A. May, and R. Horowitz, "Congested freeway microsimulation model using VISSIM," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1876, no. 1, pp. 71–81, 2004.
- [147] H. Xinghua and Z. Yu, "A parameters calibration method in simulated complex traffic network," *The Open Construction and Building Technology Journal*, vol. 9, no. 1, pp. 262–265, 2015.
- [148] N. Raju, S. Arkatkar, and G. Joshi, *Innovative Research in Transportation Infrastructure*, Springer, Singapore, 2019.
- [149] N. T. Ratrou and S. M. Rahman, "A comparative analysis of currently used microscopic and macroscopic traffic," *The Arabian Journal for Science and Engineering*, vol. 34, no. 1B, 2009.
- [150] S. L. Jones Jr., A. J. Sullivan, N. Cheekoti, M. D. Anderson, and D. Malave, "Traffic simulation software comparison study," Univ. Transp. Cent. Alabama Rep. 02217, University of Alabama, Tuscaloosa, AL, USA, 2004.
- [151] M. Azam, O. C. Puan, S. A. Hassan, and N. Mashros, "Calibration of microsimulation model for tight urban diamond interchange under heterogeneous traffic," *IOP Conference Series: Materials Science and Engineering*, vol. 527, p. 12077, 2019.
- [152] A. Saha, S. Chandra, and I. Ghosh, "Modeling platoon dispersion at signalized intersections in mixed traffic scenario," *Arabian Journal for Science and Engineering*, vol. 44, no. 5, pp. 4829–4838, 2019.
- [153] A. Mathew and T. V. Bajpai, *SiMTraM-Simulation of Mixed Traffic Mobility-User Documentation*, IIT Bombay, Mumbai, India, 2012.
- [154] P. Hidas and P. Wagner, "Review of data collection methods for microscopic traffic simulation," in *Proceedings of the 10th World Conference on Transport Research WCTR*, Istanbul, Turkey, July 2004.
- [155] M. P. Slinn Mike and P. Guest, *Traffic Engineering Design*, Elsevier, Amsterdam, Netherlands, 2005.